



The course
theme:

Methods and means of measurement

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**Lecture 1
Metrology and
standardization**



What is metrology?

Metrology science of:

- About measurements of physical quantities
- Methods and means of their unity
- Achieving the required accuracy

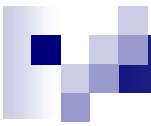
Tasks of metrology:

- establishment of units of physical quantities
- standards and reference measuring instruments
- Development of measurement theory
- ensuring the uniformity of measurements
- Methods for estimating errors
- transfer of standards to working measurements



What is metrology?

- The study of measurements
- Measurements are quantitative observations; numerical descriptions
- Measurements are part of the daily routine in labs and everywhere
- Measurements are expected to be “good”



- A “good” measurement is one that can be trusted when making decisions
- Decisions are made daily on whether measurements are good enough, but they are made subconsciously and often by different people
- Decisions need to be conscious and consistent.



Metrology Vocabulary

- Unit of measurement
- Accuracy
- Precision
- Standards
- Calibration
- Verification
- Traceability
- Tolerance
- Errors
- Uncertainty

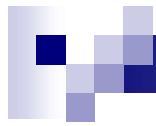


Measuring instrument - a technical means (or their complex) intended for measurements, having normalized metrological characteristics, reproducing and (or) storing a unit of physical quantity.

Standards - In metrology, a standard is an object, system, or experiment that bears a defined relationship to a unit of measurement of a physical quantity

Measures - measure on a set is a systematic way to assign a number to each suitable subset of that set, intuitively interpreted as its size.

Standard gauges are measures measuring devices or transducers intended for testing and calibration on them other means of measurement and adopted as a model.



Workers measuring instruments - measures gauges, transducers, devices, and systems used for practical measurements with scientific research, production, trade, and other fields.

Verification scheme for measuring instruments is a normative document establishing the subordination of measuring instruments participating in the transfer of the unit size from the standard to the working measuring instruments.



Measurement methods:

- **Direct measurements** - measurements taken from the data obtained after the experiment / lab.
- **Indirect measurements** - are taken based on known dependences on sizes which are subject to direct measurements.
- **Absolute measurements** - direct measurements with the use of physical constants.
- **Relative measurements** - values are compared with the same name, which is taken as the original.
- **Direct estimation method** - a method of measurement in which the value of the quantity determined directly by means of showing the measurements.
- **Method of comparison with measure, opposition, zero** - method, the measurements of which are based on comparison / comparison with standards.



Measurement methods:

- **Differential method** - are based on the solution of the boundary-layer equations in their partial-differential equation form.
- **The method of coincidences** - in metrology, a measurement technique involving comparison with a standard.
- **Element method** - is the most widely used method for solving problems of engineering and mathematical models
- **Complex method** - total quality readings.
- **Sensitivity** - the quality or condition of being sensitive.
- **Stability** - a property that expresses the consistency of impressions.
- **Control points** - points on which rely on measurements.
- **Control zone** - in which control takes place.



Measurement methods:

- **Scale length** - the distance between the axes (or centers) of two adjacent marks of the scale, measured along an imaginary line passing through the middle of the shortest marks of the scale.
- **The scale division price** - is the difference between the values corresponding to two adjacent marks of the scale.
- **Calibration characteristic** - by the calibration characteristic we understand the functional dependence of the analytical signal on the true value of the informative parameter.
- **The reading range** - is the range of scale values.
- **The measurement range** - is the range of values within which the permissible error limits of the measuring instrument are normalized.

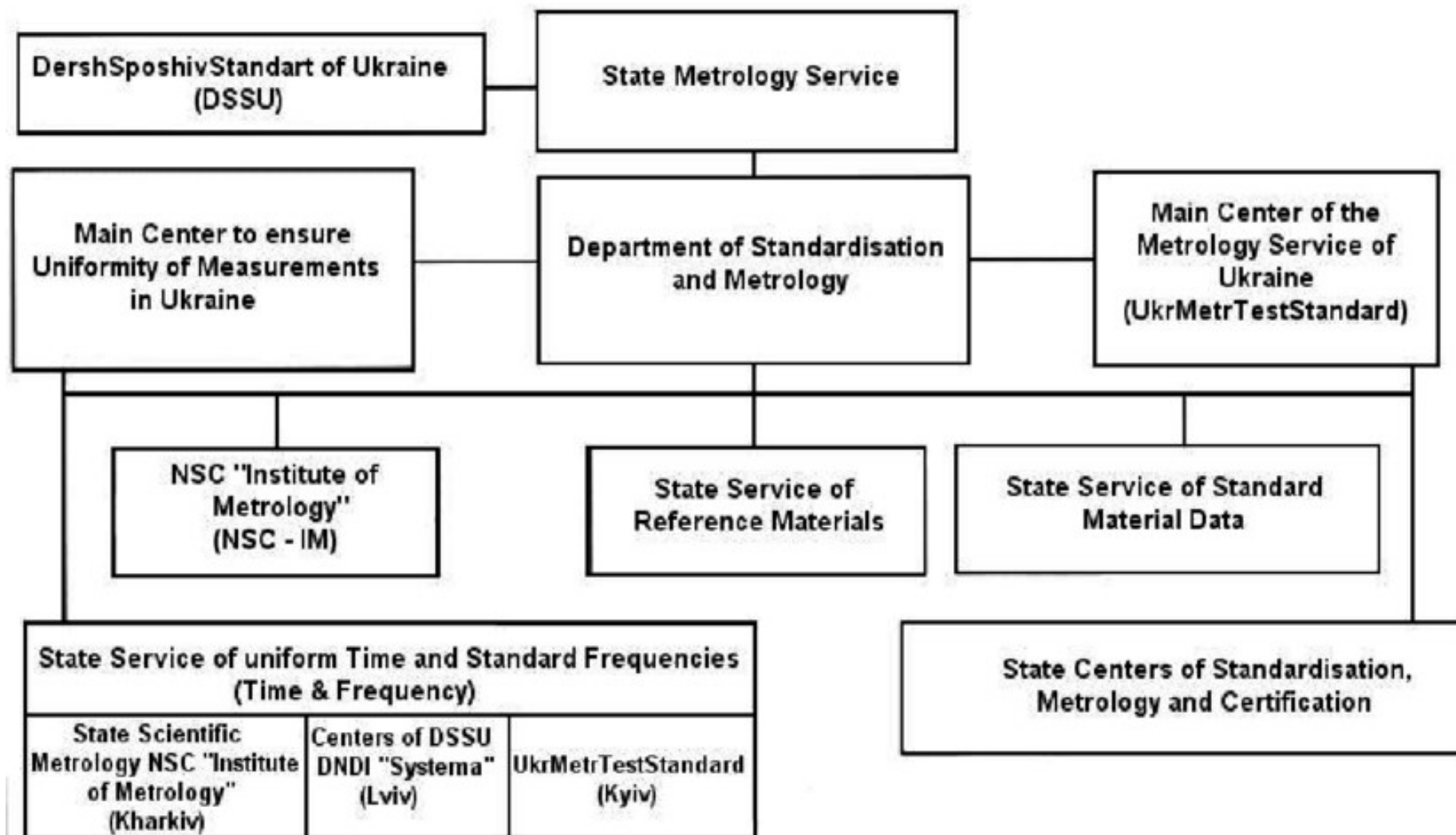


Measurement methods:

- **Influencing physical quantity**- a physical quantity that affects the size of the measured quantity and (or) the measurement result.
- **Normal conditions** - are physical conditions determined by pressure $p = 0.1013 \text{ MPa} = 760 \text{ mm Hg. Art.}$ (normal atmosphere) and a temperature of 273.15 K (0°C) at which the molar volume of the gas is $V_0 = 2.2414 \times 10^{-2} \text{ cubic meters per mol.}$

Structure of standardization and metrology services

Structure of the State Metrology Service in Ukraine





The main tasks of standardization services

- **State standard** in metrology (the science of measurement), a standard (or etalon) is an object, system, or experiment that bears a defined relationship to a unit of measurement of a physical quantity. Standards are the fundamental reference for a system of weights and measures, against which all other measuring devices are compared.
- **The Ministry** should systematically check compliance with the requirements established in the standards
- **Technical Committee** a committee whose purpose is the organization and implementation of standardization work in certain areas of activity.
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- **Certified laboratories** research and design departments of standardization of NNI, KB and at the enterprises.



Principles of standardization

- **Systematic and complexity** - These aspects significantly increase productivity and product quality.
- **Optimality** - a rational system of standards that covers all vital cycles.
- **Interrelationship** - Property of machines, devices, mechanisms to equivalent replacement of their parts or completely unit
- **Preference** - a principle that increases the level of interchangeability and reduces the range of products.



Standardization by non-geometrical parameters

Spring force

- G - Shear modulus
- d - wire diameter
- D₀ - average spring diameter
- λ - H-H₁
- H - the length of the spring
- i - the number of working turns

Relative force deviation

Absolute deviations of force from individual parameters

$$\Delta P_d = 0.25 \quad \text{N}$$

$$\Delta P_i = 0.03 \text{N}$$

$$\Delta P_{D_0} = 0.08 \text{N}$$

$$\Delta P_H = 0.11 \text{N}$$

$$\Delta P_G = 0.01 \text{N}$$



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Lecture 2
**Basic concepts: metrology,
physical quantity,
measurement scales**



Metrology -


the science of
measurements, methods
and means of ensuring
their unity and ways to
achieve the required
accuracy.



PHYSICAL QUANTITY

Any object (object, process, phenomenon) can be characterized by its properties or qualities, which are manifested to a greater or lesser extent and, therefore, are quantified.

- **Physical quantity** - a property that is common in qualitative terms in many physical objects, processes, phenomena, but individual in quantitative terms in each of them.
- **Kind of physical quantity** - qualitative certainty of physical quantity.

- 
- **The size of a physical quantity** - the quantitative content in a given material object of a property corresponding to the concept of "physical quantity".

Physical quantities that are the same in qualitative manifestation can be combined into categories, classes. The physical quantities of one such category are called **homogeneous**.

For example, physical quantities: diameter of a product, height, width, length, distance between details are homogeneous and belong to one category - length. Homogeneous quantities have the same dimension, but the same dimension is not a sign of homogeneity. Example: Energy, Work, Amount of heat - dimension $\text{m}^{-2} \text{kg c}^{-2}$ (Joules).



THE VALUE OF PHYSICAL QUANTITY

- **The numerical value of a physical quantity** is the number N_x equal to the ratio of the size of the measured physical quantity X to the size of the unit of measurement or multiple (lower) unit q_x .

$$N_x = E \left\lfloor \frac{x}{q_x} \right\rfloor$$

where $E \parallel$ - the whole part of the size ratio.

- **The value of a physical quantity** is a reflection of a physical quantity in the form of a number of units accepted for it.

$$x = N_x [q_x]$$

where $[q_x]$ is the designation of the unit of measurement.



The true value of a physical quantity is the value of

X_{true} ,

which would ideally reflect the property of the object under study.

**A real value of a physical quantity or
a conditionally true value**

is a value of a physical quantity, found experimentally and so close to the true value that it can be used instead for this purpose.

Absolute accuracy will never be achieved. So the true meaning can be forgotten: it still can not be determined.

Instead, we will use the term "**real**" value. This value, which can be determined experimentally most accurately at this stage of development of measuring technology.

SYSTEMS OF PHYSICAL QUANTITIES, DIMENSION

A system of physical quantities is a set of interconnected quantities, in which some quantities are taken as independent, and others are determined depending on them.

A basic physical quantity is a physical quantity that is part of a system of physical quantities and is accepted as independent.

Derivative physical quantity is a physical quantity that is part of a system of physical quantities and is determined through the basic quantities of this system.

The dimension of a physical quantity is an expression that displays the relationship of a physical quantity to the basic quantities of the system.

The dimension of a basic physical quantity is a conditional symbol of a physical quantity in a given system of quantities. In general, for each physical quantity, the dimensions can be written using the symbols:

$$z = L^{\alpha} M^{\beta} T^{\gamma} I^{\varepsilon} \theta^n J^{\lambda} N^{\delta}$$

where z is the measured physical quantity; L, M, T, I, θ, J, N - physical quantities, taken as basic (L - length, M - mass, T - time, I - current, θ - temperature, J - light intensity, N - quantity substances);

$\alpha, \beta, \gamma, \varepsilon, n, \lambda, \delta$ are indicators of the degree to which the principal quantity enters the equation when determining the derivative quantity.



DIMENSIONAL AND DIMENSIONLESS PHYSICAL QUANTITIES

- **Dimensional physical quantity** - a quantity in the dimension of which at least one of the basic quantities is raised to a degree not equal to zero.
- **Dimensionless physical quantity** - a quantity in the dimension of which the main quantities are included in the degree equal to zero.

The ISO recommend the following types of dimensionless quantities:

- **Factor** - the coefficient of linear dependence of two quantities of the same dimension, but inhomogeneous.

Example: power factor (power factor); $\cos\varphi = P/S$; quality factor.

- **Relation** - the result of dividing two homogeneous quantities

Example: reflection ratio.

- **Relative quantity** - the ratio of the value to the reference homogeneous.

Example: relative magnetic permeability.

- **Level** is the logarithm of a relative quantity.

Example: incoherent units are introduced for the level - bell...



UNITS OF MEASUREMENT

A unit of measurement is a physical quantity of a certain size, accepted by agreement for the quantitative display of homogeneous quantities.

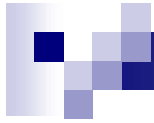
Examples of quantities taken by our ancestors as units

The golden belt of the Kyiv grand duke Svyatoslav is a unit of length, about which the ancient deed (1073) says: "It's the measure and base" (its length was 108 cm).

Yard - the distance from the tip of the nose of English King Henry I (XI-XII centuries) to the end of the middle finger of his outstretched hand (According to other sources - the length of the sword of the same king).

The legal inch - the length of three barley grains - was established by English King Edward II in 1324.

Carat - the mass of the seed of one of the species of beans.



A system of units of measurement is a set of basic and derived units belonging to a certain system of quantities.

Basic unit - a unit of basic quantity.


Derivative unit - a unit of derivative quantity.

An extrasystem unit is a unit that is not part of any system of units.

A coherent unit is a derivative unit related to other units of the system by an equation in which the numerical coefficient is equal to one.

Coherent system of units - a system of units, all derivative units of which are coherent.

The International System of Units (SI) is a coherent system of units adopted and recommended by the General Conference on Weights and Measures.



But wherever and with whatever accuracy the measurement is carried out, directly or indirectly, it is always based on the comparison of the measured quantity with its concrete realization, taken as a unit. And the same physical quantity, the same parameter must be expressed by any device in the same legalized unit. All over the globe. This is the demand of our time. Choosing units of physical quantities is not an easy task.

How to measure the length of a boa? (folktale)

Animals found several boa-independent units. The length of the snake was equal to the length of 38 parrots and one parrot's wing, or the length of 5 monkeys, or the length of 2 elephants.

The International System of Units (SI), adopted in the vast majority of countries around the world, is the result of many years of work by scientists in many countries. Metrology is essentially an international science that requires constant international cooperation of its specialists - metrologists.



SCALES OF MEASUREMENTS

When considering measurements in a broad sense, as measurements of physical and non-physical quantities, the following four scales of measurement are distinguished.

The scale of names (or nominal) is the simplest. The numbers on this scale are just shortcuts for distinguishing and detecting objects being studied, such as team players. There are no units or measures of comparison in this scale. The numbers that make up the scale of names are assigned arbitrarily, they can be swapped, but cannot be added or subtracted. Based on established traditions, the use of a scale of names should not be considered a dimension.

The order scale is also called rank or non-metric. Ranks are places occupied in the scale of order, in ancient times - titles, ranks. In sports, ranks are places taken in competitions, or the results of ranking athletes by a group of experts. According to the ranks, it is possible to make judgments such as "better - worse", "more - less", characteristic of control. Ranks determine qualitative rather than quantitative indicators. There are also no units of measurement.



The scale of intervals differs from the scale of order in that the numbers (ranks) are separated by well-defined intervals.

A feature of this scale is the arbitrary choice of the starting point (zero point).

Examples of using the interval scale are setting the calendar time (date), measuring the temperature, measuring the joint angle.

The use of the interval scale refers to measurement (both in the broad and narrow sense of the term).

Finally, **a scale of relations** that does not impose any restrictions on the rules of mathematical processing of the results obtained when using it.

In this scale, the position of the zero point is strictly defined. This is how - with a fixed start - we measure time intervals, distances, force, etc., comparing the results with the second, meter, kilogram and other units of physical quantities.



**Thank you for your
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Lecture 3
Basic concepts:
measurement, types of
measurements, means of
measurements



MEASUREMENT -

display of measured values by their values by experiment and calculations by means of special technical means.




Measurements can be classified:

- according to the characteristics of accuracy - *equal, non-equal*;

Equivalent measurements - a series of measurements of any quantity, performed with the same accuracy of measuring instruments and in the same conditions.

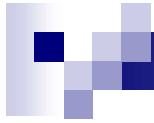
Non-equilateral measurements are a series of measurements of any quantity performed by several measuring instruments of different accuracy and (or) in different conditions.

- 
- by the number of measurements in a number of measurements - *single, multiple*

Single measurement is a measurement performed once.

Sometimes two, three or more measurements of the same value are performed to be more confident in the obtained result. However, only one of them is used as a result, and the measurement is considered one-time.


Multiple measurement - a measurement of the same physical quantity, the result of which is obtained from several successive measurements, ie. consisting of a series of single measurements.



- **in relation to change of the measured size -**
static, dynamic;

Static measurement - a measurement of a physical quantity taken in accordance with a specific measurement task as unchanged during the measurement time.

Dynamic measurement is the measurement of a changing physical quantity.

- 
- according to the expression of the measurement result - *absolute, relative*;

Absolute measurement - the concept of absolute measurement was introduced with the following definition: a measurement based on direct measurements of one or more basic quantities and (or) the use of physical constants.

Relative measurement - the measurement of the ratio of a quantity to a quantity of the same name, which plays the role of a unit, or a change in a quantity in relation to a quantity of the same name, taken as the original.

- 
- **by general methods of obtaining measurement results -**
direct, indirect, joint, aggregate.

Direct measurement is the measurement of a quantity without transforming its kind and using known dependencies.

Indirect measurement - the measurement of a quantity with a transformation of its kind or with calculations based on the results of measurements of other quantities, with which the measured value is associated with a clear functional dependence.

Aggregate measurements - simultaneous measurements of several homogeneous quantities, in which the desired values of the quantities are determined by solving a system of equations obtained by measuring different combinations of these quantities.

Joint measurements - simultaneous measurements of several disparate quantities to determine the relationship between them.



MEASUREMENT RESULT

is the value of a physical quantity found by measuring it.

The measurement result is always accompanied by a characteristic of its uncertainty.

- ***Absolute measurement error*** is the difference between the measurement result and the true value of the measured quantity.

$$\Delta = x - x_{true}$$

- ***Relative measurement error*** - the ratio of the absolute measurement error to the true value of the measured value.

$$\delta = \frac{\Delta}{x_{true}} = \frac{x}{x_{true}} - 1$$

Relative error is represented either as relative units or as a percentage.




MEASUREMENT PROCEDURE. MEASURING OPERATIONS

- **Measurement procedure** is a sequence of measurement operations that provides measurement according to the selected method.
- **Measuring operation** - an operation with physical quantities or their values during the measurement process.

The minimum set of measurement operations for the implementation of the measurement procedure is: *the reproduction and comparison operations*.

Reproduction (physical quantity) - a measuring operation consisting in the creation and (or) storage of a physical quantity of a given value.

Comparison (physical quantities) - a measurement operation consisting in reflecting the relationship between the sizes of two homogeneous physical quantities by the corresponding conclusion: more, less or equal in size.



If the sample and measured physical quantities are not consistent in size and genus, there is a need for another measurement operation - measurement transformation.

Measuring transformation (physical quantity) - a measuring operation in which the input physical quantity is converted into an output functionally related to it.

The principle of measurement transformation is called the physical effect on which it is based.

Measurement transformations are divided into measuring with a change of the kind of quantity and without change of the kind of quantity, which, in turn, are divided into linear and nonlinear.

A scale measurement transformation is a linear measurement transformation of an input quantity without a genus transformation.



MEASURING EQUIPMENT (ME)

- a technical tool used in measurements and having standardized metrological characteristics.

ME includes: *measuring instruments* and *measuring devices*

- ***Measuring instruments*** implement the measuring procedure completely, and measuring devices - only one measuring operation.
- ***Measuring device*** - a means of measuring equipment, which performs only one of the components of the measurement procedure (measuring operation).




Types of measuring devices:

■ ***Measure*** –

a measuring device that implements the reproduction and (or) storage of the physical value of a given value. (Examples: dashed measure of length, quartz oscillator - a measure of the frequency of electrical oscillations)

Measures are subdivided into: *unambiguous*, *multivalued*, *sets of measures*, *stores of measures*. In addition, there may be imported measures and built-in measures.



An *unambiguous measure* is a measure that reproduces the physical quantity of one value. An unambiguous measure can be regulated or unregulated.

A *multivalued measure* is a measure that reproduces a physical quantity with different values.

(Example: dashed measure of length.)

A *set of measures* is a set of measures of different values of the same physical quantity, necessary for application in practice, both individually and in various combinations (Example: set of weights.)

Measure store - a set of measures, structurally combined into a single device, in which there are devices for connecting them in various combinations.



Types of measuring devices:

- **Comparator** - a measuring device that compares homogeneous physical quantities.
- **Measuring transducer** - a measuring device that implements a measuring transformation
- **Scale transducer** - a transducer designed to change the value an integer number of times (voltage divider)
- **Primary transducer, sensor** - a transducer that first interacts with the object of measurement.
- **Computing component (measuring instruments), numerical measuring transducer** - a measuring device that is a set of computer equipment and software that performs computational operations during measurement.



MEASURING INSTRUMENTS (MI)

- a means of measuring equipment that implements the measurement procedure.

Include: *code MI, recording MI, gages, measuring channels (of systems), measuring systems.*

MI implement the measurement procedure and create a signal of measuring information. Depending on the addressee, the measurement information signal can be visual (intended for a human operator) and code (intended for perception by technical devices.) According to the type of measurement information signal, measuring instruments are divided into: *gages, code measuring instruments and recording measuring instruments.*


- 
- **Gage** is a mean of measurement in which a visual signal of measuring information is created.

Measuring instruments can be *analog* and *digital*.

Analog gage - a measuring instrument in which the visual signal of the measuring information is represented by a scale and a pointer.

Digital gage - a measuring instrument in which the visual signal of the measuring information is represented in the form of numbers or symbols on the indicating device.

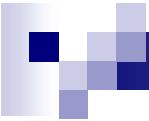
- **Code measuring instrument** (analog-to-digital converter) is a measuring instrument in which a code signal of measuring information is created.
- **Measuring recording means** - measuring means in which the signal of measuring information is registered.



Measuring instruments and measuring devices can be combined into measuring systems.

Measuring system - a set of measuring channels, measuring devices and other technical means combined to create signals of measuring information about several measured values.

- ***Measuring channel*** - a set of measuring equipment, communication means and other technical means, designed to create a signal of measuring information about one measured physical quantity.
- ***Measuring information system*** - a set of SIT, means of control, diagnostics and other technical means combined to create signals of measuring and other types of information.



Devices or substances, which create a signal of information (indicators) do not belong to MI, although some MI can be used as indicators.

Computer equipment is widely used in modern MI.

The use of computer technology and modern intelligent information technology was the basis for the creation of such modern MI, which was called - ***intelligent measuring equipment (IME)***.

IME - a tool for measuring equipment, containing databases and knowledge that are used both when performing the measurement procedure in full and when performing individual measurement operations using a system of optimizing rules, decision-making, determining their own behavior, controlling their performance, self-learning and communicating with people -operator.



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Lecture 4
Basic concepts:
measurement methods,
measurement errors



METHODS OF PHYSICAL QUANTITIES MEASUREMENT

Analysis of measurement methods is one of the main sections of metrology.

The purpose of each measurement is to obtain a result in the form of the value of a physical quantity.

Peculiarities of obtaining the result are taken into account when analyzing the properties of measuring equipment:

measures, comparators, measuring transducers.



Classification and analysis of direct measurement methods

The method of measurements is one of the basic generic concepts of metrology, and to obtain a complete picture, the methods of measurement must be strictly defined, classified in accordance with the essential classification features.


Measurement method - a set of techniques for using measuring equipment and the principle of measurement to create a signal of measuring information.

The principle of measurement is the scientific basis of the measurement method.

- The first feature of direct measurement methods is the number of stages of using measuring equipment.

In accordance with this feature, the methods of direct measurements are divided into:

one-stage (comparisons); multi-stage (balancing).

- 
- The second feature in the analysis of direct measurement methods is the use of single or multi-digit measures and scale transducers.

When ***single-stage methods (comparisons)*** implementing,
are used:

- a multi-valued unregulated measure (MUM);
- a multi-channel comparator (C) or a single-valued non-adjustable measure (SNAM);
- a multi-digit scale converter (MDSC);
- a comparator.

When ***multi-stage methods (balancing)*** implementing, are used:

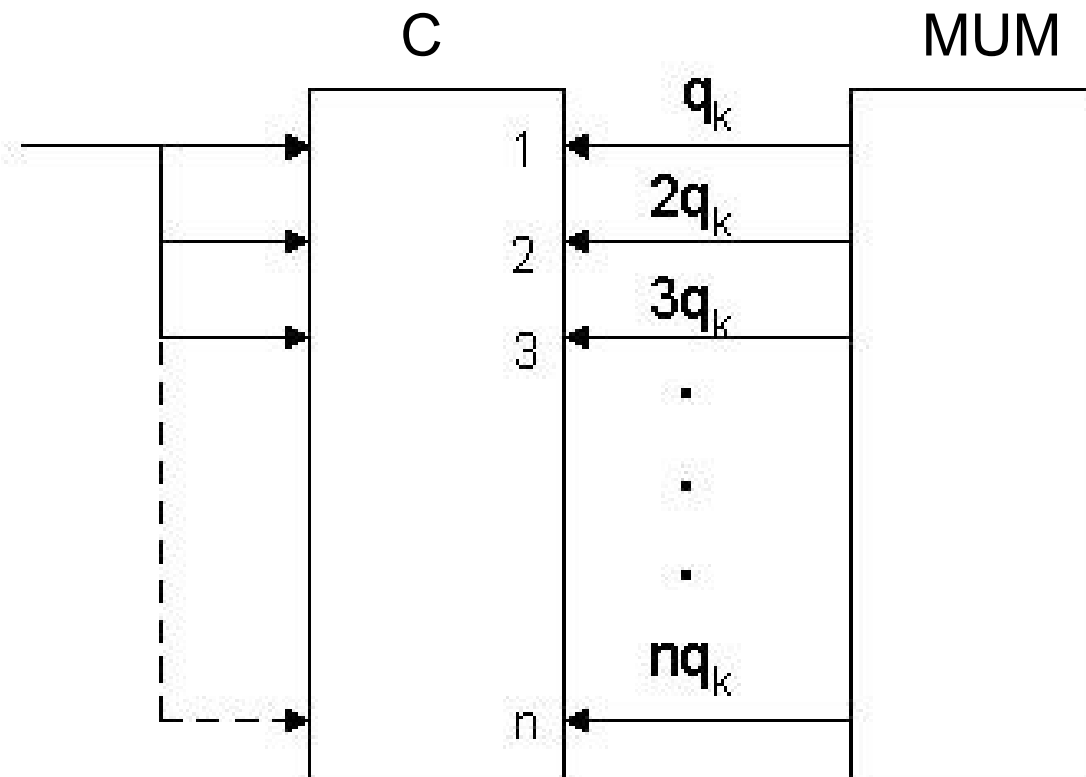
- a unique adjustable measure (UAM);
- a comparator or an unambiguous non-adjustable measure (UNAM);
- an adjustable scale converter (ASC);
- a comparator.

Method of comparison

is a method of measurement with simultaneous comparison of the measured quantity with all output values of a multi-valued non-adjustable measure.

This method provides the maximum speed of measurement, for example: length, electric tension, mechanical sizes.

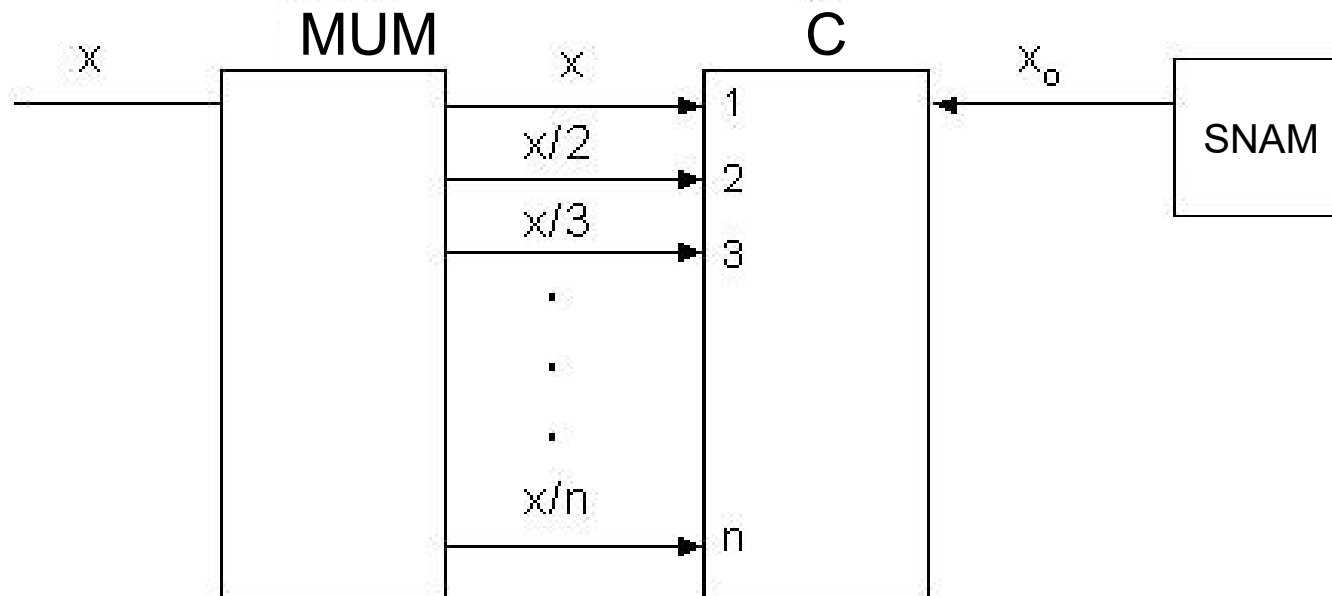
Diagram of the measuring instrument, implementing this method:




The measurement result is determined in accordance with the channel number of the comparator i , for which the difference $|x - iq_K|$ is minimal, where q_K is the degree of quantization (division price) of the MUM. The equation of measurement according to the scheme has the form:

$$x = iq_K = E \left| \frac{x}{q_K} + 0,5 \text{sign} x \right|$$

The second variant of the comparison method can be implemented using a multi-valued scale converter MUM:





The measurement result is determined in accordance with the channel number of the comparator i , for which the difference $\left| \frac{x}{i} - x_0 \right|$ is minimal, where x_0 is the sample value reproduced by a unique non-adjustable measure. The equation of measurement according to the scheme has the form:

$$x = ix_0 = E \left| \frac{x}{x_0} + 0,5 \operatorname{sign} x \right|$$

The disadvantage of this method is as follows: the characteristic of each MDSC channel is actually the division coefficient $K_{di} = \frac{1}{i}$ in which case the relationship between x and Kd will be nonlinear

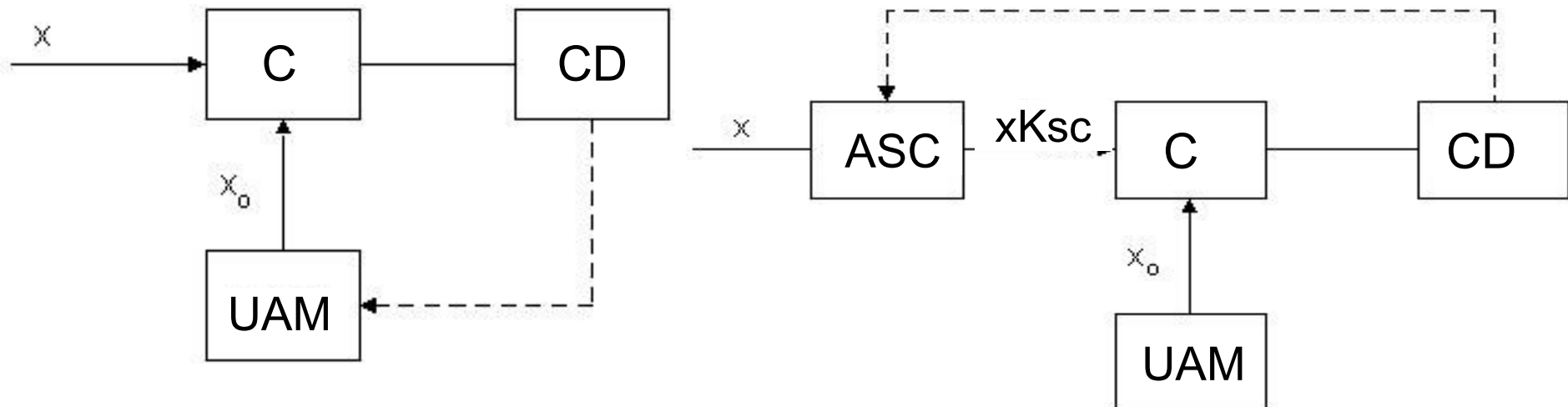
$$x = \frac{x_0}{K_{di}}$$

Balancing method

When implementing the balancing method, an unambiguous adjustable measure (UAM) or an adjustable scale converter (ASC) can be used.

Balancing method with adjustable measure - a method of measurement with a comparison of the measured value and the value that is reproduced by the measure adjustable before their complete balancing.

Balancing method with adjustable scale converter (SC) - a method of measurement comparing the value that is reproduced by a unique non-adjustable measure, and the measured value that went through the adjustable scale converter, the conversion factor of which changes until complete equilibration of the compared values.



CD - Control Device




The Nonius method

The disadvantage of the comparison method is that the measurement error implementing this method cannot be less than half of the minimum division of a multivalued unregulated measure. To increase accuracy, use hardware redundancy, ie. instead of one multivalued measure, two multivalued measures are used, the price of division of which differs slightly. Consider an example of the Nonius method.

Assume that you need to measure the length of the product x , which is less than the division length of the multivalued measure q_K . Initial reference points of item length and measure are combined and to length x add the second multivalued measure with the division length $q_K \left(1 - \frac{1}{n}\right)$.

Find the point of approximate coincidence of two quantities iq_K and $iq_K \left(1 - \frac{1}{n}\right)$:

$$iq_K = iq_K \left(1 - \frac{1}{n}\right) + x$$



From here we get $x = i \cdot \frac{q_x}{n}$. Thus, when applying two multivalued measures, we obtain the same effect as when applying one multivalued measure with division length $\frac{q_x}{n}$.

The number ***n*** is usually chosen 10; 100; 1000, etc.

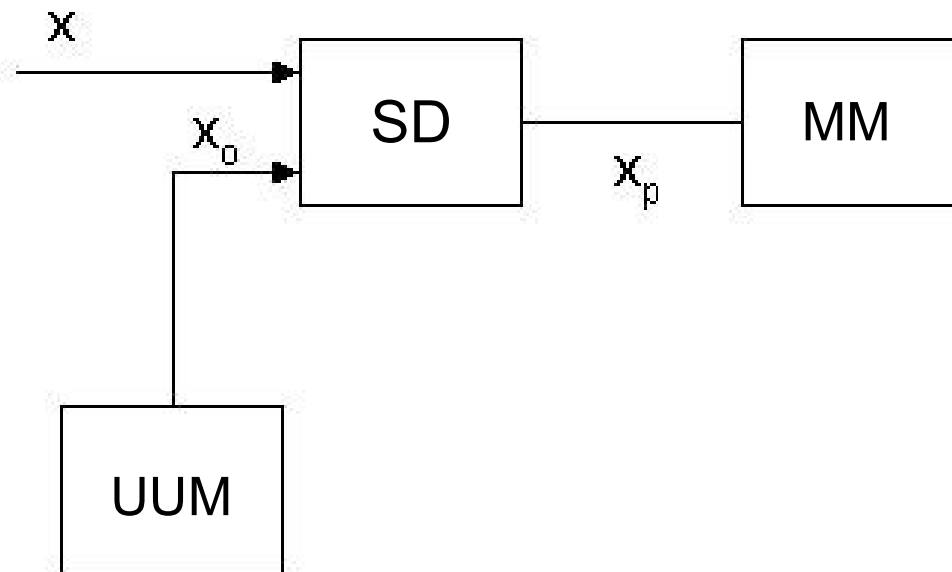
Example:

Thus, the Nonius method is a method of measurement with finding the coincidence of numerical values of two multi-valued unregulated measures, the zero marks of which are shifted by the measured value.

Differential measurement method

is a measurement method in which a small difference between the measured value and the output value of an unambiguous non-adjustable measure is measured by an appropriate measuring instrument.

The block diagram implementing the differential method is shown:



Where: MM - means of measurements, UUM is an unambiguous unregulated measure, SD is a subtraction device.

According to the indications of MM get the value of the difference x_p ,
then

$$x = x_0 + x_p$$

Based on the definition of this method, the difference should be small.

The reason for this: The absolute error of measurement x consists of the absolute error of the output quantity and measure $\Delta(x_0)$ and the absolute error of measurement of the difference $\Delta(x_p)$, ie.

$$\Delta(x) = \Delta(x_0) + \Delta(x_p)$$

The relative error $\delta(x)$ of measurement x is equal to:

$$\delta(x) = \frac{\Delta(x)}{x} = \frac{\Delta(x_0)}{x} + \frac{\Delta(x_p)}{x}$$

The first component of the error under the condition of proximity

x and x_0 is equal $\delta(x) = \frac{\Delta(x_0)}{x} \approx \frac{\Delta(x_0)}{x_0}$

$$\frac{\Delta(x)}{x} = \frac{\Delta(x_p)}{x_p} \cdot \frac{x_p}{x} = \delta(x_p) \cdot \frac{x_p}{x}$$

The second component is equal:

Where $\delta(x_p)$ is the relative error of MM. Thus, if the difference is small, ie. $x_p \ll x$, the measurement error caused by the inaccuracy of MM measuring the difference is significantly reduced.



Substitution method

According to the principle of operation, is similar to the *method of balancing* with the difference that the measured value and the output value of the measure act at the input of the measuring instrument at the same time.

Thus, at the MM input is first affected by the measured value, and then it is turned off and replaced by the output value of the unambiguous adjustable measure (UAM). Then the value of the output value of the measure (subject to complete substitution), determine the value of the measured value.

Knowing the classification of methods will allow you to choose one or another measurement method for the optimal solution of the measurement problem.

And if you chose the right measurement method, performed it with an accurate measuring instrument, but did not get the true value of the measured quantity, as each result contains some measurement error - the deviation of the measurement result from the true value of the measured value.

It is necessary to know the classification of errors in order to determine their type and use the methods developed by metrology to exclude them.



Measurement accuracy

is the main characteristic of the quality and perfection of measurements. It is characterized by the degree of closeness of the measurement result to the true value of the measured value.

An indicator of measurement accuracy is the measurement error.

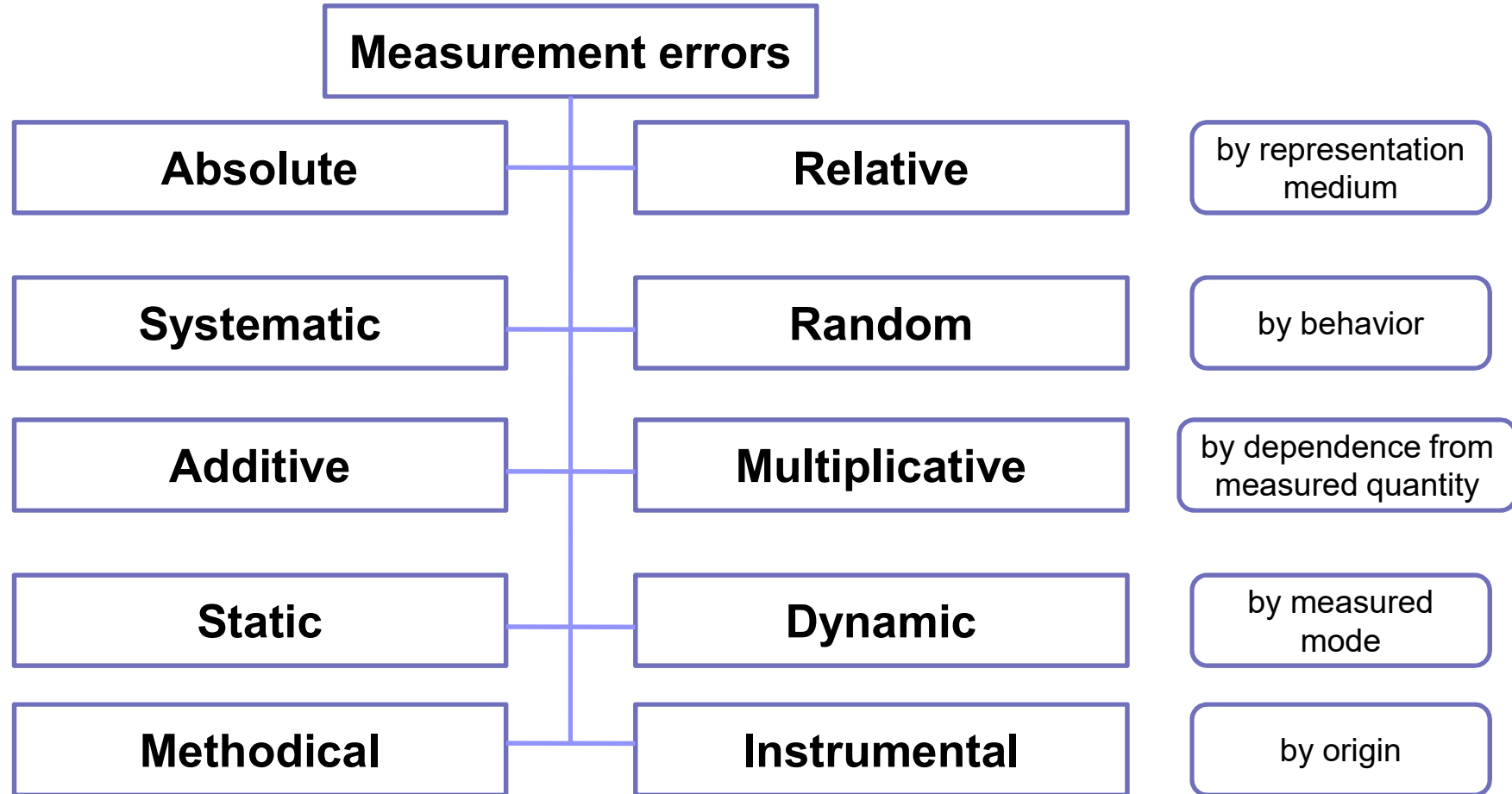
The accuracy of the measurement must meet the purpose of the measurement.

Excessive accuracy leads to unreasonable costs.

But insufficient measurement accuracy can lead to losses.

Systematization of measurement errors

Errors of measurement and means of measuring equipment are systematized in accordance with their features:



Which determine their main features.



Absolute and relative errors

Absolute error is the measurement error that is expressed in units of the measured quantity. The absolute error is equal to the difference between the value of x and the true value of x_{true} :

$$\Delta = x - x_{true}$$

If the actual value is used instead of the true value, then:

$$\Delta = x - x_{actual}$$

The error Δ is an indicator of measurement accuracy.

Relative error - the ratio of absolute error to the true value of the measured value.

Relative error is expressed in relative units or as a percentage.

$$\delta = \frac{\Delta}{x_{true}} = \frac{x - x_{true}}{x_{true}} = \frac{x}{x_{true}} - 1$$



Systematic and random errors

- **Systematic error**

is a component of measurement error, which either remains constant and changes according to a certain law during repeated measurements.

- **Random error**

is a component of error that changes unpredictably (randomly) when repeated measurements of the same value.



Additive and multiplicative errors

Depending on the absolute error of the measured value x errors are divided into additive and multiplicative:

- **Additive error** is a component of absolute error that does not depend on the measured value.
- **Multiplicative error** is a component of absolute error, which linearly depends on the measured value. If the dependence of the absolute error on the measured value is more complex than the linear one, the nonlinear components of the error are subject to analysis.



Static and dynamic errors

According to the mode of measurement, errors are divided into static and dynamic:

- **Static error** - the error of the static measurement, ie. an error that occurs at a constant measure quantity.
- **Dynamic error** - a component of the error, which occurs in addition to static in dynamic measurements, ie. when changing the measured value.




Methodical and instrumental errors

Methodical measurement error is a component of measurement error, which is due to the inadequacy of the object of measurement of its model, adopted during the measurement and transmission errors.

Example:

An example of an error due to the inadequacy of the model is the measurement of the amount of gasoline in the tank of the aircraft Q at the level of h . The exact analytical dependence is unknown and is assumed to be linear $h = k \cdot Q$, which is the cause of methodological error.



Instrumental error is a component of measurement error due to the imperfection of the measuring instrument. The instrumental component of the error is due to the imperfection of the design and manufacturing technology of measuring equipment (ME), deviation from the nominal value and time instability of the parameters of elements and nodes of ME, sensitivity to external influences and uninformative parameters of the input signal, interaction of ME with the object of measurement. The instrumental error consists of:

Measurement error due to interaction - a component of the instrumental error, which occurs due to the influence of the measuring instrument on the state of the object of measurement.

Measurement error (absolute) - the difference between the reading of measuring instrument (MI) and the true value of the measured value in the absence of methodological error and error due to the interaction of MI with the object of measurement. Includes:

- **The main error** is the MI error under normal conditions of use.
- **Additional error** is the MI error when the values of the influencing values deviate from normal within the operating conditions of use.



**Thank you for your
attention!**





The course
theme:

Methods and means of measurement

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Lecture 5
**Characteristics of random
errors**



Introduction

In science, the word "error" does not have the usual meaning of something wrong.


"Inaccuracy" in scientific measurement refers to the inevitable inaccuracy that accompanies all measurements.

Errors, as such, cannot be attributed to the blunders of the experimenter.

You cannot avoid them by trying to be very attentive.

The best you can count on is to keep errors as low as possible and to calculate their value reliably.

Knowledge of the distributions of random errors allows this to be done.



Difference between the measurement result x and the true value of the measured quantity x_{true} is called **the absolute error of the measurement result**:

$$\Delta = x - x_{true}$$

The error is a random variable. It can be represented as:

$$\Delta = \Delta_s + \overset{\circ}{\Delta}$$


where Δ_s - expected error Δ ;

$\overset{\circ}{\Delta}$ - random error with zero mathematical expectation.

Non-random quantity Δ_s called systematic error, and $\overset{\circ}{\Delta}$ - random error.

If the values Δ_s is known, the systematic error can be eliminated by taking the final measurement result; x_{cor} - corrected measurement result

$$x_{cor} = x - \Delta_s$$



A random error cannot be ruled out, since it is not known what specific value the random variable $\overset{\circ}{\Delta}$ took for a given measurement. To assess the influence of a random error on the measurement result, set positive Δ_1 , negative Δ_2 and find the probability that the true value of the measured quantity lies between $x - \Delta_2$ and $x + \Delta_1$. Interval $[x - \Delta_2; x + \Delta_1]$ called **the confidence interval**, and the probability that x_{cor} is within this interval with a confidence level P_c .

It can be shown that

$$P_c = P\left[-\Delta_1 \leq \overset{\circ}{\Delta} \leq \Delta_2\right] \text{ (formula 1)}$$

Usually choose $\Delta_1 = \Delta_2$. Then

$$P_c = P\left[|\overset{\circ}{\Delta}| \leq \Delta_1\right]$$



Example of Confidence probability and Confidence Interval Estimating

Suppose we need to weigh a 100 samples, for which we have a balance that allows us to determine the mass with an error of 0.05 g (for example, due to the fact that the smallest weight used for weighing has a mass of 0.1 g).

The maximum load allowed by the balance does not allow placing more than one item to be weighed on the cup.

We know that at each weighing the error can be either positive or negative, not exceeding the absolute value of 0.05 g.

It is natural to assume that we will be wrong equally often both in the direction of understating and in the direction of overstating the weight.

Then the interpretation of formula (1) for the given case has the following form

$$P_{c_1} = P\left[0 \leq \overset{\circ}{\Delta} \leq 0,05\right] = 0,5 = 50\% \quad (\text{when the mass is overstated});$$

$$P_{c_2} = P\left[-0,05 \leq \overset{\circ}{\Delta} < 0\right] = 0,5 = 50\% \quad (\text{if the weight is underestimated}).$$

Distributions of random errors

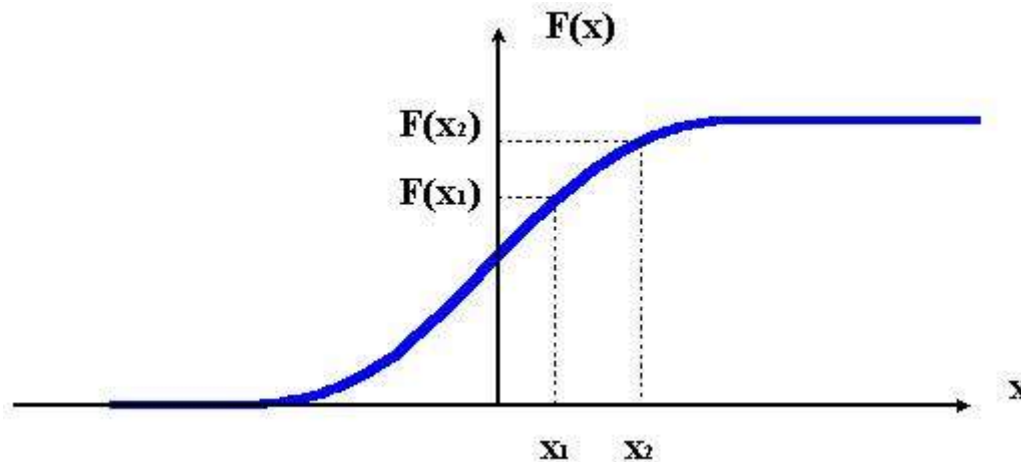
To characterize random errors, the so-called probability distribution function of a random variable is used x , defined by equality $F(x) = P(X < x)$

Let us indicate some of the main properties of this function.

■ Property 1

Function $F(x)$ not decrease with increasing x

The probability distribution function of a random variable x



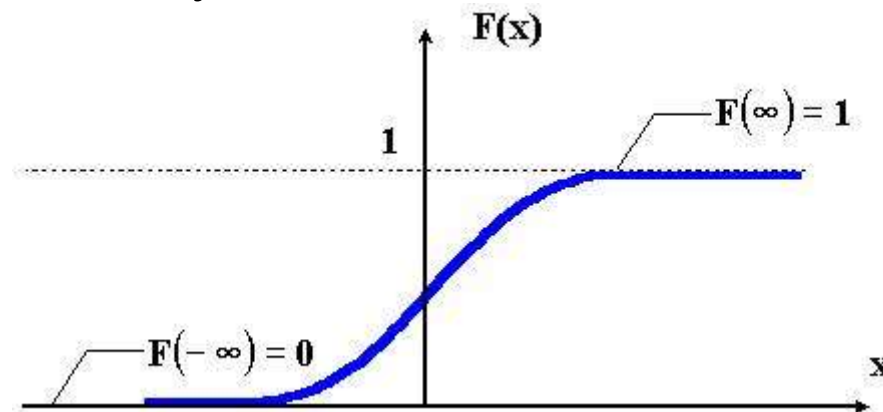
Indeed, if x_1 and x_2 - some real numbers, and $x_2 > x_1$, then difference $F(x_2)$ and $F(x_1)$ is equal to the probability of finding the quantity x in the interval $[x_1, x_2]$, and the probability is always greater than zero (or in the extreme case is zero if $x_2 = x_1$).

$$F(x_2) - F(x_1) = P(x_1 \leq X \leq x_2) \geq 0$$

■ Property 2

The probability distribution function for $x = -\infty$ is zero, the probability distribution function at $x = \infty$ equal to 1. $F(-\infty) = 0$;
 $F(\infty) = 1$

The probability distribution function of a random variable:



We proceed to consider the probability density of a random variable, which is denoted $p(x)$. In a number of problems, of interest is the probability that a random variable is located in the interval $x \leq X \leq x + \Delta x$. This probability can be found as the difference $P(x \leq X \leq x + \Delta x) = F(x + \Delta x) - F(x)$

This probability depends not only on the value x , but also on the length Δx considered interval. The quantity $p(x)$, equal

$$p(x) = \lim_{\Delta x \rightarrow 0} \frac{P(x \leq X \leq x + \Delta x)}{\Delta x}$$

it is customary to call the *probability density of a random variable*.

From the above dependences it directly follows that the distribution density is the derivative of the distribution function, and the distribution function is the density integral:

$$P(x) = \frac{dF(x)}{dx} \quad F(x) = \int_{-\infty}^x P(u) du \quad \int_{-\infty}^{\infty} p(x) = F(\infty) = 1$$

It can be easily shown that:

Those. the area under the probability density curve is equal to one.

If we use the designation of the absolute error, then the probability distribution function is denoted $F(\Delta)$, and the probability density - $p(\Delta)$.

The function or density of the distribution of the random error are the most complete probabilistic characteristics of this error. However, in solving many theoretical and applied problems, point and interval characteristics of random errors are widely used. The most commonly used dotted characteristics include the mathematical expectation $M[\Delta]$, dispersion $D[\Delta]$ and the standard deviation $\sigma[\Delta]$. The mathematical expectation $M[\Delta]$ of random error is determined by the expression:

$$M[\Delta] = \int_{-\infty}^{\infty} \Delta \cdot p(\Delta) d\Delta$$

The error model $M[\Delta] = \Delta_s$, and $M\left[\overset{\circ}{\Delta}\right] = 0$ (the random error is centered).



The dispersion of the random error is determined by the expression:

$$D[\Delta] = \int_{-\infty}^{\infty} (\Delta - M[\Delta])^2 \cdot p(\Delta) d\Delta = \int_{-\infty}^{\infty} (\Delta - \Delta_s)^2 \cdot p(\Delta) d\Delta$$

Dispersion characterizes the spread of error values relative to its mathematical expectation. The use of dispersion $D[\Delta]$ in practice is inconvenient, since its dimension differs from the dimension of the measured value. Therefore, instead of $D[\Delta]$ often using the standard deviation $\sigma[\Delta]$, determined by the expression:

$$\sigma[\Delta] = \sqrt{D[\Delta]}$$

having the same dimension as the measured value.

Normal distribution of errors (Gaussian distribution)

The function and density of this distribution are determined by the expressions:

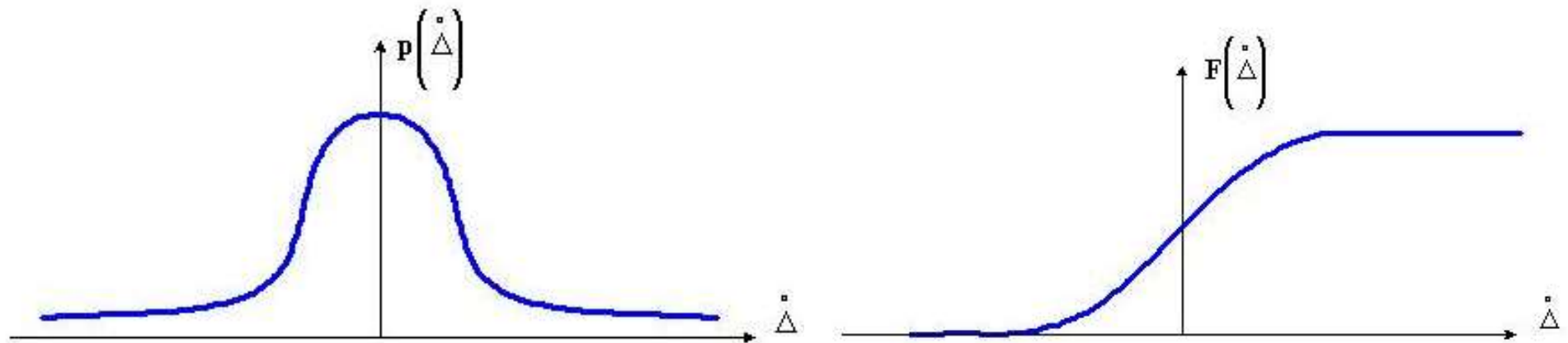
$$F(\Delta) = \frac{1}{2} + \Phi\left(\frac{\Delta - \Delta_s}{\sigma}\right) \quad p(\Delta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{(\Delta - \Delta_s)^2}{2\sigma^2}\right)$$

where $\Phi\left(\frac{\Delta - \Delta_s}{\sigma}\right) = \Phi(Z)$ - is Laplace function, which is tabulated.

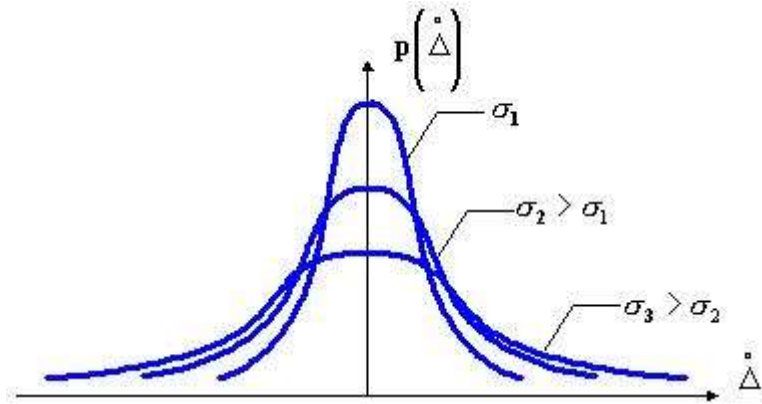
If $\Delta_s = 0$

$$F\left(\overset{\circ}{\Delta}\right) = \frac{1}{2} + \Phi\left(\frac{\overset{\circ}{\Delta}}{\sigma}\right) \quad p\left(\overset{\circ}{\Delta}\right) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{\overset{\circ}{\Delta}^2}{2\sigma^2}\right)$$

Graphs $F\left(\overset{\circ}{\Delta}\right)$ and $p\left(\overset{\circ}{\Delta}\right)$ are shown:



Dotted characteristic - the standard deviation σ determines the shape of the distribution.



Normal distribution at different σ

Figure shows the graphs of the probability density function for different values σ . If

σ decreases, the ordinate at zero increases. The rise of the curve in the central part is compensated for by its sharper fall towards the abscissa axis, since the area under the probability density curve remains unchanged and equal to 1.

At very low values σ , almost all of the area under the curve is concentrated in a small interval centered at zero. With increasing σ , the curve "flattens", taking on an increasingly flat-topped shape. The estimation of the confidence interval and the probability of falling into the confidence interval when is $\Delta_s \neq 0$ carried out in accordance with the expression

$$P_c = P[-\Delta_1 \leq \Delta \leq \Delta_2] = \left[\Phi\left(\frac{\Delta_2 - \Delta_1}{\sigma}\right) + \Phi\left(\frac{\Delta_1 + \Delta_2}{\sigma}\right) \right] \quad \Phi(-z) = -\Phi(z)$$

$$\text{If } \Delta_s = 0, \quad \text{then} \quad P_c = P[-\Delta_1 \leq \Delta \leq \Delta_2] = \left[\Phi\left(\frac{\Delta_2}{\sigma}\right) + \Phi\left(\frac{\Delta_1}{\sigma}\right) \right]$$

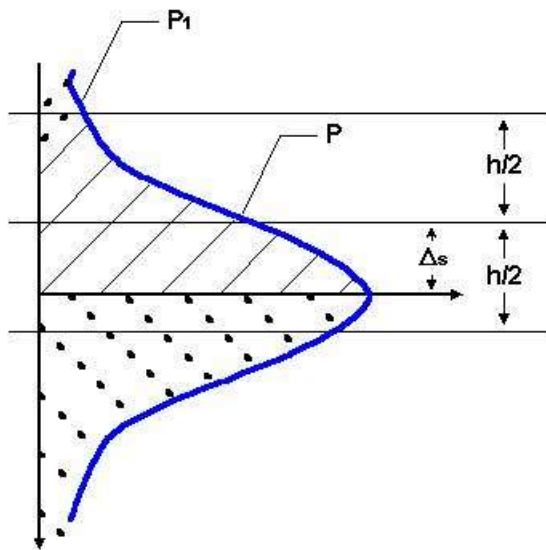
$$\text{If } \Delta_1 = \Delta_2 = \Delta, \text{ then } P_c = P[-\Delta_1 \leq \Delta \leq \Delta_2] = 2\Phi\left(\frac{\Delta_1}{\sigma}\right)$$

EXAMPLE

The systematic error of a device designed to maintain desired aircraft altitude, equal to $\Delta_s = -20\text{m}$ and random error is the standard deviation $\sigma[\dot{\Delta}] = 50\text{ m}$. The aircraft is assigned a corridor with a height of $h = 100\text{ m}$. Find the probability that the plane will fly lower, higher, and inside the corridor if the plane is given an altitude that corresponds to the middle of the corridor, and the random errors of the device are normally distributed.

Decision

Figure shows the distribution of errors of the instrument for maintaining the height of the aircraft. The mathematical expectation is equal to the value of the systematic error Δ_s . Therefore, the center of grouping of random errors is shifted from the middle of the corridor downward by $= -20\text{mA}$. The figure shows that the probability of flight inside the corridor is



$$P = \Phi\left[\frac{h/2 + \Delta_s}{\sigma[\dot{\Delta}]}\right] + \Phi\left[\frac{h/2 - \Delta_s}{\sigma[\dot{\Delta}]}\right] = \Phi\left(\frac{30}{50}\right) + \Phi\left(\frac{70}{50}\right) = \Phi(1,4) + (0,6).$$

Using the Laplace functions, we obtain: $F(1.4) = 0.41924$; $F(0.6) = 0.22575$. Then $P = 0.645$

The probability that the plane will fly above the corridor is

$$P1 = \frac{1}{2} - \Phi \left[\frac{h/2 + \Delta_s}{\sigma[\dot{\Delta}]} \right] = \frac{1}{2} - \Phi \left(\frac{70}{50} \right) = 0.081$$

The probability that the plane will fly below the corridor is

$$P2 = \frac{1}{2} - \Phi \left[\frac{h/2 - \Delta_s}{\sigma[\dot{\Delta}]} \right] = \frac{1}{2} - \Phi \left(\frac{30}{50} \right) = 0.5 - 0.226 = 0.274$$

To solve the problem, it is possible to use the following relationships. To determine the probability $P(\Delta_1 < \Delta < \Delta_2)$ being in the range Δ_1, Δ_2 of accidental error, distributed normally, in the presence of systematic error Δ_s is determined interval

$$P(\Delta_1 < \Delta < \Delta_2) = \Phi \left[\frac{\Delta_2 - \Delta_s}{\sigma[\dot{\Delta}]} \right] - \Phi \left[\frac{\Delta_1 - \Delta_s}{\sigma[\dot{\Delta}]} \right]$$

$$P(-\Delta_\Gamma < \Delta < \Delta_\Gamma) = \Phi \left[\frac{\Delta_\Gamma - \Delta_s}{\sigma[\dot{\Delta}]} \right] + \Phi \left[\frac{\Delta_\Gamma + \Delta_s}{\sigma[\dot{\Delta}]} \right]$$

If $\Delta_2 = -\Delta_1 = \Delta_\Gamma$,

then

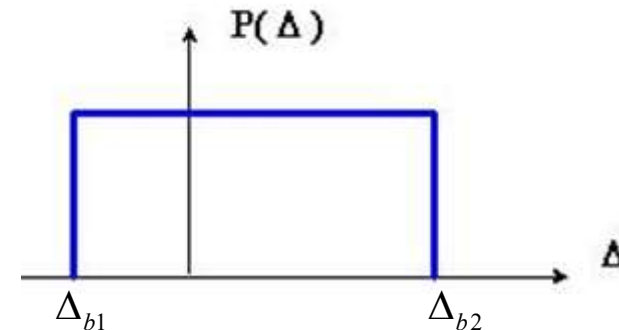
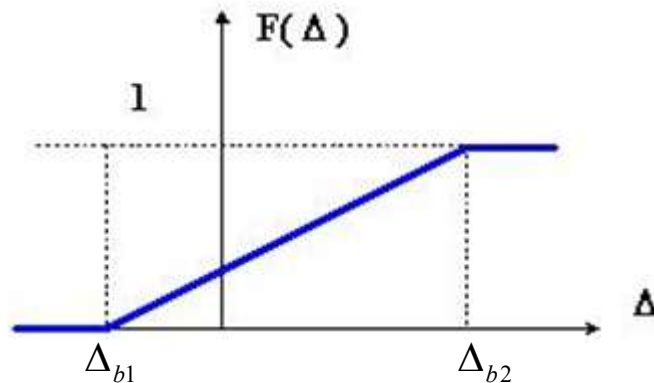
To find the probability of a flight inside the corridor, we use the last relation provided that $\Delta_\Gamma = \frac{h}{2}$

$$P(-h/2 < \Delta < h/2) = \Phi \left[\frac{h/2 - \Delta_s}{\sigma[\dot{\Delta}]} \right] + \Phi \left[\frac{h/2 + \Delta_s}{\sigma[\dot{\Delta}]} \right]$$

Uniform distribution of errors

With a uniform distribution, the random error Δ with the same probability can take any value in the interval $\Delta_{b1} \leq \Delta \leq \Delta_{b2}$, where Δ_{b1}, Δ_{b2} the boundary values of the error. The function $F(\Delta)$ and density $p(\Delta)$ of this distribution are determined by the expressions

$$F(\Delta) = \begin{cases} 0, & \text{given } \Delta < \Delta_{b1} \\ \frac{\Delta - \Delta_{b1}}{\Delta_{b2} - \Delta_{b1}}, & \text{given } \Delta_{b1} \leq \Delta \leq \Delta_{b2} \\ 1, & \text{given } \Delta > \Delta_{b2} \end{cases} \quad p(\Delta) = \begin{cases} 0, & \text{given } \Delta < \Delta_{b1} \\ \frac{1}{\Delta_{b2} - \Delta_{b1}}, & \text{given } \Delta_{b1} \leq \Delta \leq \Delta_{b2} \\ 0, & \text{given } \Delta > \Delta_{b2} \end{cases}$$



The dispersion of the uniform distribution is $D[\Delta] = \frac{(\Delta_{b2} - \Delta_{b1})^2}{12}$,

and the standard deviation is

$$\sigma[\Delta] = \frac{\Delta_{b2} - \Delta_{b1}}{2\sqrt{3}},$$



**Thank you for your
attention!**





The course
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**Lecture 6
Distributions of random
errors**



The most complete characteristic of a random error as a random quantity x is a function of probability distribution $F(\Delta) = P(x < \Delta)$

where Δ is some current value of the error.

The distribution function $F(\Delta)$ sometimes called the ***cumulative distribution function***.

For random variables with a continuous and differentiable distribution function, you can find the differential distribution or ***the probability density distribution function***, expressed as a derivative of $F(\Delta)$,

$$p(\Delta) = \frac{dF(\Delta)}{d\Delta}$$

The probability density is a dimensional quantity

$$\dim p(\Delta) = \dim \frac{1}{\Delta}$$

Dotted characteristics of random errors

To obtain ***dotted characteristics of the error***, ***moments*** are used. If we denote by $m_k[\Delta]$ and, $\mu_k[\Delta]$ respectively, the k -th initial moment and the k -th central moments of the error, then

$$m_k[\Delta] = \int_{-\infty}^{\infty} \Delta^k \cdot p(\Delta) \cdot d\Delta; \quad \mu_k[\Delta] = \int_{-\infty}^{\infty} (\Delta - M[\Delta])^k \cdot p(\Delta) \cdot d\Delta$$

The following are used as dotted characteristics of the error:

- **the first initial moment** is the *mathematical expectation*

$$m_1[\Delta] = M[\Delta] = \int_{-\infty}^{\infty} \Delta \cdot p(\Delta) \cdot d\Delta$$

- **second central moment - dispersion** $\mu_2[\Delta] = D[\Delta] = \int_{-\infty}^{\infty} (\Delta - M[\Delta])^2 \cdot p(\Delta) \cdot d\Delta$

The physical meaning of dispersion is the power of dissipation relative to the mathematical expectation (standard deviation)

$$\sigma[\Delta] = \sqrt{D[\Delta]}$$

The third and fourth central moments are used to characterize the shape of the distribution:

The coefficient of asymmetry of the distribution is $\gamma_a[\Delta] = \frac{\mu_3[\Delta]}{\sigma^3[\Delta]}$

- The skewness coefficient $\gamma_a[\Delta]$ is zero for symmetric distributions.
- To characterize the slope of the distribution are used: - kurtosis: $\mathfrak{Z} = \frac{\mu_4[\Delta]}{\sigma^4[\Delta]}$
- - kurtosis coefficient $\gamma_{\mathfrak{Z}}$, $\gamma_{\mathfrak{Z}} = \mathfrak{Z} - 3$
- - counterexcess

$$\mathfrak{x} = \sqrt{\frac{\mu_3[\Delta]}{\sigma^3[\Delta]}} = \frac{1}{\sqrt{\mathfrak{Z}}}$$

To estimate the characteristics of the error, the **coordinate of the distribution center is used**. If the coordinate of the distribution center is Δ_y found from the symmetry condition, that is, at $F[\Delta_y] = 0,5$, then the resulting value Δ_y is called the **median**. This characteristic can be used for the class of symmetric exponential distributions. If the coordinate of the distribution center is found as the center of gravity, then such an abscissa is equal to the mathematical expectation.

$$\Delta_y = \int_{-\infty}^{\infty} \Delta \cdot p[\Delta] \cdot d\Delta = M[\Delta]$$



Interval characteristics of random errors

In the theory of probability, mathematical statistics and, accordingly, in the theory of errors, the concepts of “ **quantile** ”, “ **interquartile interval** ”, “ **confidence probability** ”, “ **level of significance** ” etc. are used.

a% quantile is the abscissa of the probability density distribution, to the left of which is a% of the area of the distribution curve.

The interquartile gap is the difference between the (100-a)% and a% quantiles. Between the verticals of the symmetric centered distribution, which bound the interquantile interval, there is (100-2a)% of the distribution curve area.

The confidence probability P_{conf} is the probability of finding a random error Δ in the permissible zone within the confidence limits Δ_{b1} and Δ_{b2} :

$$P(\Delta_{r1} < \Delta < \Delta_{r2})$$

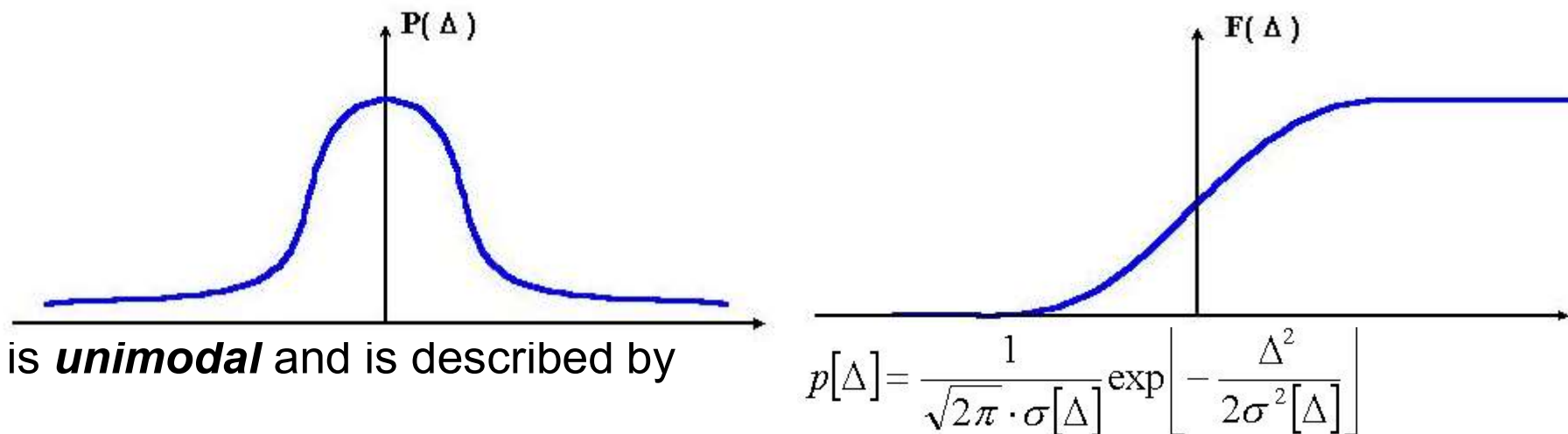
The level of significance $\alpha = 1 - \alpha$ is the probability that a random variable X is in the interval between $\alpha\%$ and $+\infty$. The critical area is characterized by the level of significance. If the found estimate is in the critical region, then this hypothesis is not accepted. If the level of significance is overestimated, the correct hypothesis may be rejected, and if the level is underestimated, an incorrect hypothesis may be accepted. Therefore, the level of significance is usually taken in the range (10 ... 2.5)%.

Normal distribution

Properties: - **symmetry**; - **monotonic decrease in the probability density**.

If a random error is a composition of more than four random, independent and equal errors, then approximately it obeys the normal law, regardless of the distribution of the components. Measurement errors usually consist of several random and independent components and therefore are normally distributed in most cases.

The normal distribution of a centered random variable:



Where $\sigma[\Delta]$ is the standard deviation of the random error Δ

The normalized form is obtained when $\sigma = 1$, after substitution $\Delta = \sigma \cdot z$

$$p[z] = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{z^2}{2}\right]$$



Distribution function of the error of the normalized normal distribution

$$F[z] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z \exp\left[-\frac{U^2}{2}\right] dU = \frac{1}{2} + \Phi(z)$$

If the boundaries of the confidence interval and are set , then the ***probability of falling into the confidence interval***

$$P_{\partial o\partial} = F\left(\frac{\Delta_{r2}}{\sigma}\right) - F\left(\frac{\Delta_{r1}}{\sigma}\right) = F(z_2) - F(z_1) = \Phi(z_2) - \Phi(z_1)$$

With the symmetry of the boundaries $\Delta_{r1} = -\Delta_r$ and $\Delta_{r2} = \Delta_r$ taking into account that the Laplace function $\Phi(z)$ is even, we obtain

$$P_{\partial o\partial} = 2 \cdot \Phi(z)$$

Skewness coefficient of normal distribution

$$\gamma_a = 0$$

Excess

$$E = 3$$

Kurtosis coefficient

$$\gamma_3 = 0$$

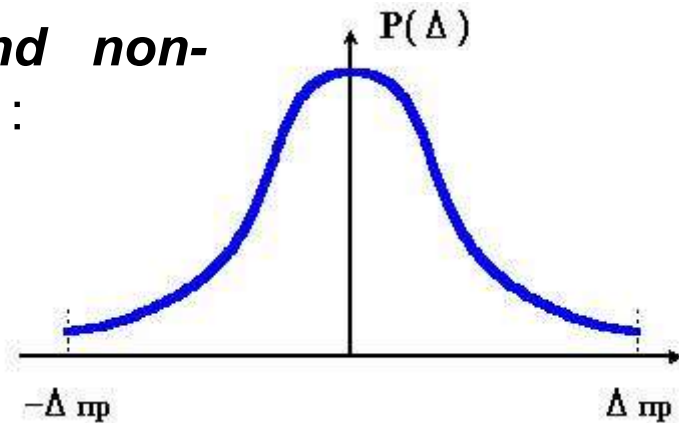
Normal Truncated Error Distribution

In practice, the **normal truncated distribution** is often used, for which $|\Delta_{np}| = 3\sigma[\Delta]$

The difference between the truncated and non-truncated normal distributions is insignificant :

For $|\Delta_r| = 2\sigma$: 0.9544 (for a normal distribution); $P = 0.9542$ (for a normal truncated);

For $|\Delta_r| = 3\sigma$: $P = 0.997$ (for normal distribution); $P = 1$ (for normal truncated).



The distribution function of the error of the normal truncated distribution has the form

$$F_y(z) = t^{-1} \left[F\left(\frac{z}{k}\right) - F\left(\frac{-3}{k}\right) \right], \text{ where } z \leq 3, t = 0.9969, k = 1.0148$$

Then the **probability of falling into the interval** $[\Delta_{r1}, \Delta_{r2}]$

$$P_{\partial os} = F_y(z_2) - F_y(z_1) = t^{-1} [F(z_2/k) - F(z_1/k)] = t^{-1} [\Phi(z_2/k) - \Phi(z_1/k)]$$

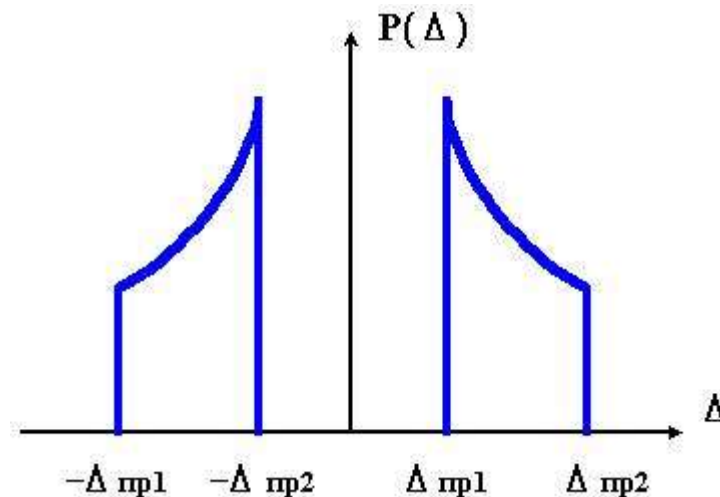
With symmetrical boundaries $P_{\partial os} = 2t^{-1} \Phi(z/k)$

• **The shape coefficients** for the normal truncated distribution are: $\gamma_a = 0$

$$\gamma_3 = 2.81, \quad \gamma_9 = -0.19$$

Bimodal distribution

is typical, for example, for deviations from the nominal value for mass products - capacitors, resistors, which are classified into classes. If for the first class the deviations are normalized in the range of $\pm 1\%$, and for the second - $\pm 2\%$, then, naturally, there will be a dip in the probability density distribution curve of the deviation of the resistors remaining after the selection of the first class in the range of $\pm 1\%$.



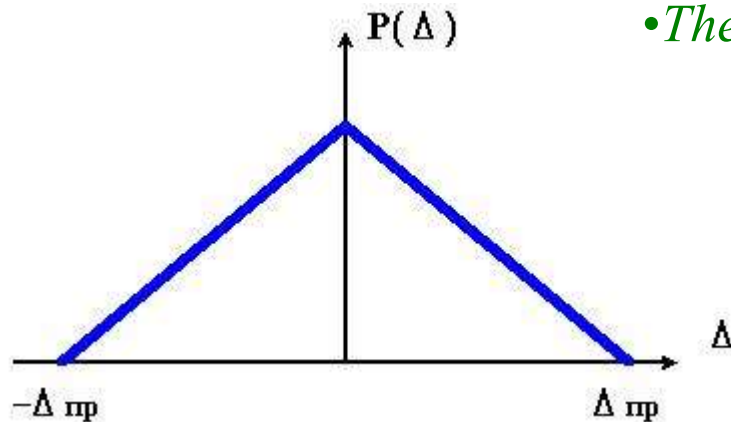
If we then select also resistors of the second class with deviations of $\pm 2\%$, then the distribution of deviations will be bimodal. It can be noted that such a distribution is possessed, for example, by the error from mechanical backlash in the transmission during alternating motion.

Triangular distribution (Simpson)

is analytically expressed by the formula:

$$p[\Delta] = \frac{1}{\Delta_{np}} \left[1 - \left| \frac{\Delta}{\Delta_{np}} \right| \right]$$

• *The distribution function* has the following form



$$F(\Delta) = \begin{cases} 0, & \Delta < 0 \\ \frac{1}{2} \left(1 + \frac{\Delta}{\Delta_{np}} \right)^2, & -\Delta_{np} \leq \Delta \leq 0 \\ 1 - \frac{1}{2} \left(1 - \frac{\Delta}{\Delta_{np}} \right)^2, & 0 \leq \Delta \leq \Delta_{np} \\ 1, & \Delta > \Delta_{np} \end{cases}$$

According to Simpson, the following errors are distributed: total when measuring lengths, angles by the difference of two rounded readings when their errors are uniformly distributed; two-count substitution measurements; total from quantization of the time interval with uniform asymmetric distributions of its two components.

• *Expected value* $M[\Delta] = 0$ • *Form factor* $\gamma_a = 0$ • *Kurtosis coefficient*

• *Standard deviation* • *Excess* $E = 2.4$, $\gamma_3 = -0.6$

$$\sigma[\Delta] = \frac{\Delta_{np}}{\sqrt{6}} \approx \frac{\Delta_{np}}{2.4}$$

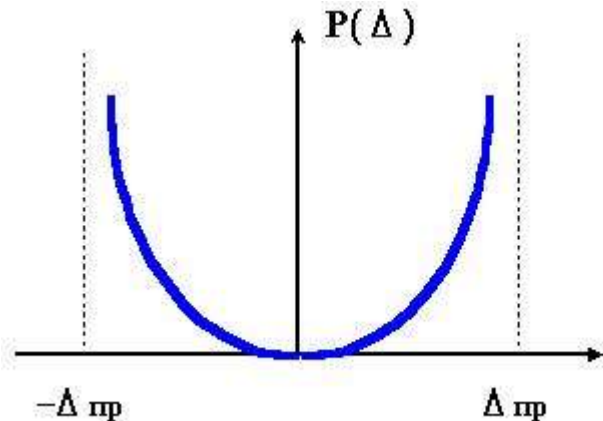
• *and %-ile and (100-a) % cent quantile distribution Simpson can be found as follows:*

$$F(\Delta_{a\%}) = \frac{1}{2} \left(1 + \frac{\Delta_{a\%}}{\Delta_{np}} \right)^2 \quad F(\Delta_{(100-a)\%}) = 1 - \frac{1}{2} \left(1 - \frac{\Delta_{(100-a)\%}}{\Delta_{np}} \right)^2$$

$$p(\Delta) = \frac{1}{\pi \Delta_{np} \sqrt{1 - \left(\frac{\Delta}{\Delta_{np}} \right)^2}}$$

Arcsine distribution

The expression for the probability density has the form:

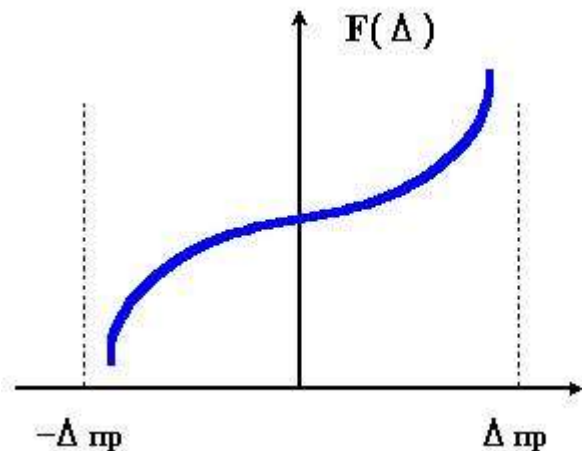


This is how the errors from sinusoidal interference, overturning of indicating instruments with supports on cores are distributed. Such a distribution can be obtained if $\Delta = \Delta_{np} \sin \varphi$

where φ is a uniformly distributed quantity.

• *The probability distribution function* has the form

$$F(\Delta) = \frac{1}{2} + \frac{1}{\pi} \arcsin \frac{\Delta}{\Delta_{np}}$$



- *and α -ile and $(100-\alpha)$ % cent quantile* for arcsine
- distribution can be found from the following equations

$$F(\Delta_{\alpha\%}) = \frac{1}{2} + \frac{1}{\pi} \arcsin \frac{\Delta_{\alpha\%}}{\Delta_{np}} \quad F(\Delta_{(100-\alpha)\%}) = \frac{1}{2} + \frac{1}{\pi} \arcsin \frac{\Delta_{(100-\alpha)\%}}{\Delta_{np}}$$

$$\Delta_{\alpha\%} = \Delta_{np} \sin \left\{ \pi \left[F(\Delta_{\alpha\%}) - \frac{1}{2} \right] \right\} \quad \Delta_{(100-\alpha)\%} = \Delta_{np} \sin \left\{ \pi \left[F(\Delta_{(100-\alpha)\%}) - \frac{1}{2} \right] \right\}$$

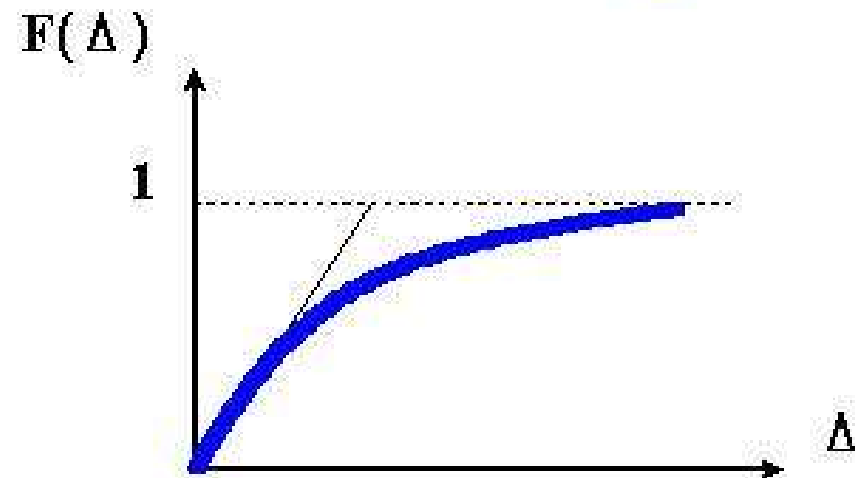
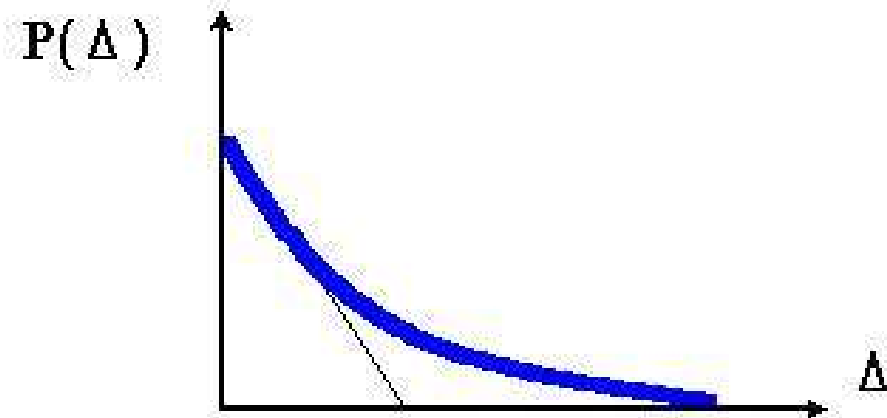
If, for example, $P = 0.95$, then the boundaries of the confidence interval are:

$$\Delta_l = \Delta_{2,5\%} = -0,997 \Delta_{np} \quad \Delta_h = \Delta_{97,5\%} = 0,997 \Delta_{np}$$

Exponential distribution of errors

The probability density function of the *exponential distribution* has the form

$$p(\Delta) = \begin{cases} \lambda \cdot e^{-\lambda \Delta} & \Delta \geq 0 \\ 0 & \Delta < 0 \end{cases}$$

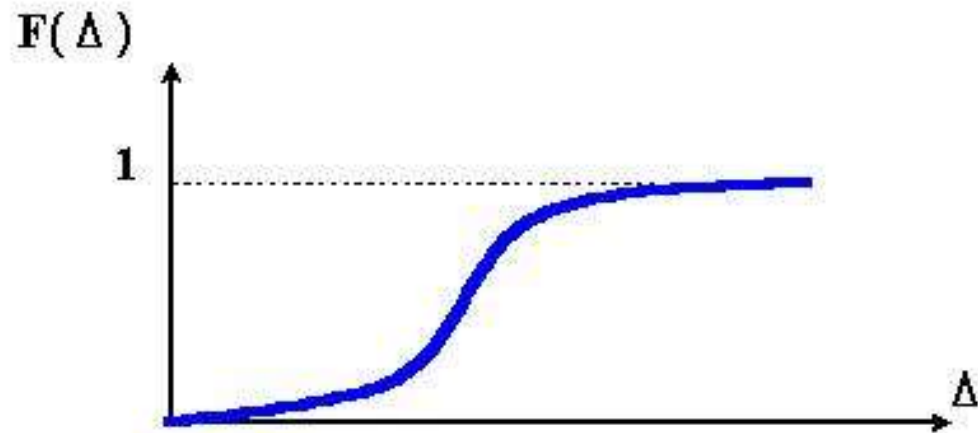
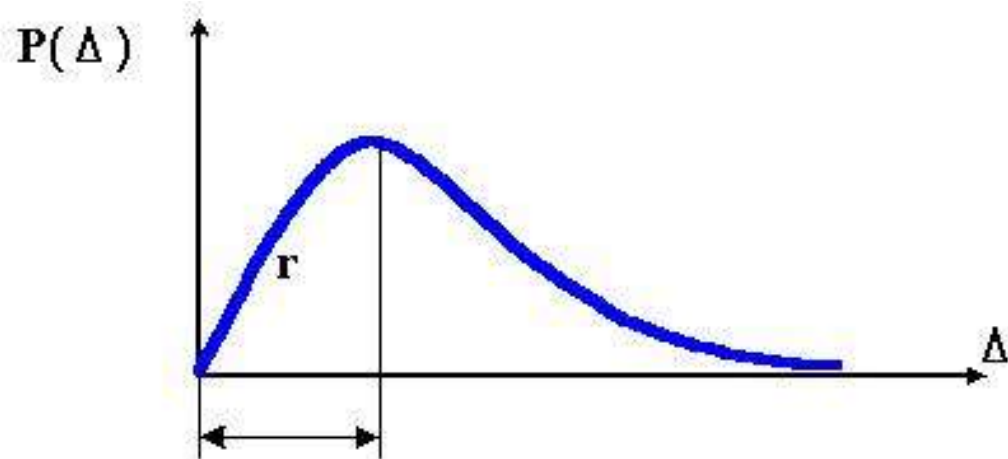


Rayleigh distribution

The probability density function of *the Rayleigh (Rayleigh-Rice)* distribution has the following form for the normalized distribution

$$p(\Delta) = \frac{\Delta}{r^2} e^{-\frac{\Delta^2}{2r^2}} ; \Delta > 0$$

where r is the distribution mode.





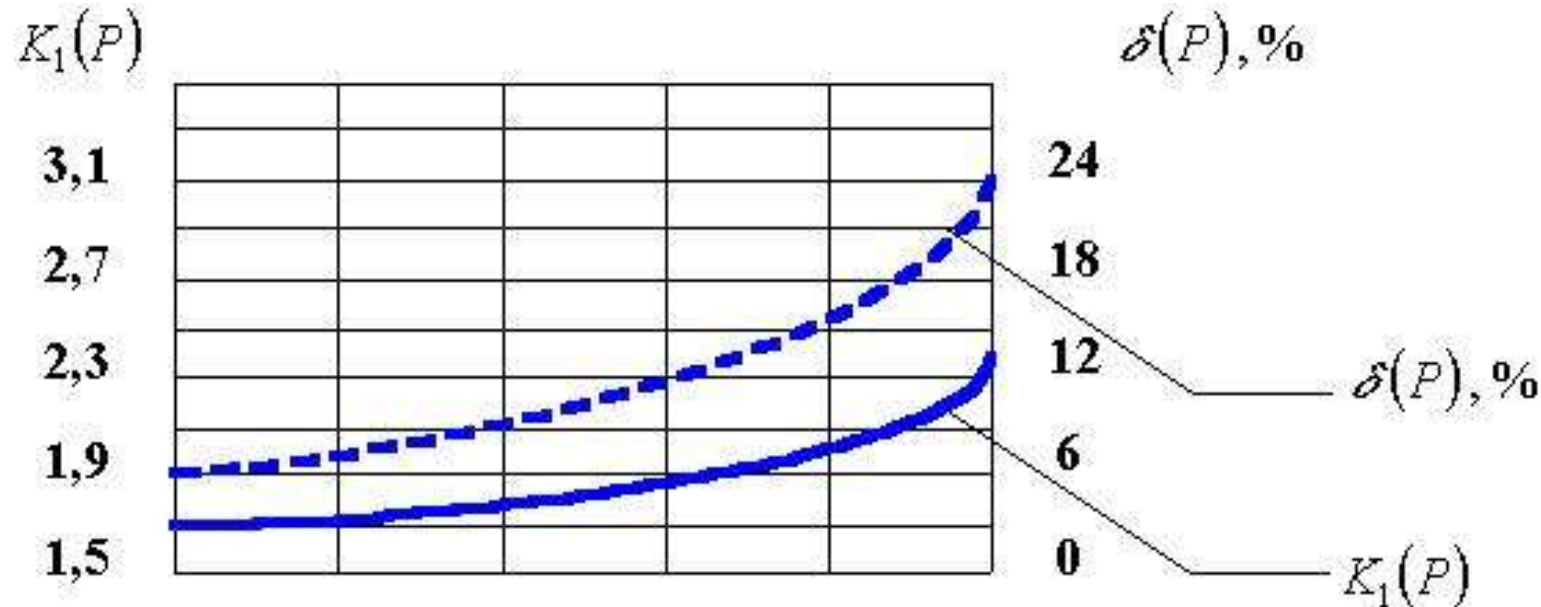
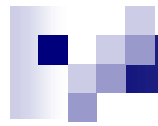
An approximate estimate of the boundaries of the confidence interval

If the probability density is symmetric, unimodal, nonzero on a finite interval of values of the argument, then the approximate value of the boundaries of the confidence interval (lower Δ_l and upper Δ_h) can be found for a given probability in accordance with the standard. In general, for a given confidence probability P , the boundaries of the confidence interval (without taking into account the sign) can be represented as

$$|\Delta_l| = |\Delta_h| = K(p) \cdot \sigma$$

where $K(p)$ is a coefficient that depends on the shape of the distribution and the value of the probability P .

For symmetric unimodal truncated distributions, some of which are considered above, an upper bound for the coefficient $K(P)$ can be found. Let's designate it $K_1(P)$. The graph of the coefficient $K_1(P) = f(P)$ and the error $\delta(P)$, % estimation error of the coefficient $K_1(P)$ is given in the appendix to the standard and in Fig



So, for example, for the probability $P = 0.95$ $K_1(P) = 1.96$

For symmetric unimodal distributions, the exact value of the coefficient **K** (**P**) is equal to the following values:

Normal distribution $P = 0.95 = 2F(z)$. $F(z) = 0.475$. $Z = 1.96$, $K_H(P) = 1.96$.

Uniform symmetric distribution $P = 0.95$

$$K_p(P) = \frac{0.95\Delta_{np}}{\sigma_p} = \frac{0.95\Delta_{np}\sqrt{3}}{\Delta_{np}} = 1.64$$

Simpson distribution $|\Delta| = \Delta_{np}$ at $P = 0.95$

$$K_c(P) = \frac{0.78\Delta_{np}}{\sigma_c} = \frac{0.78\Delta_{np}2.4}{\Delta_{np}} = 1.87$$

Thus, the graph $K_1(P)$ provides a top estimate .



**Thank you for your
attention!**





The course
theme:

Methods and means of measurement

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Lecture 7
**METHODS FOR
INCREASING THE
ACCURACY OF
MEASUREMENTS**



The theory of methods for improving measurement accuracy is an important part of metrology.

The lecture outlines general questions of methods for improving accuracy.

The purpose of the lecture is to study the classification of methods for increasing accuracy, the most common methods for correcting systematic errors and the basics of the method of statistical minimization of random errors.



Classification of methods to improve accuracy

In the methods of increasing the accuracy of measurements, two groups can be distinguished:

***methods based on the prevention of errors
(prevention);***

***methods based on the reduction of already
existing errors (treatment).***



Accuracy Improvement Techniques Based on Preventing Errors

These methods of increasing the accuracy prevent the occurrence of errors above the permissible by reducing the effect of external and changes in internal influencing quantities in a localized space and, in turn, are *classified depending on the implementation path*:

methods of stabilizing influencing quantities (thermostating, stabilization of supply voltage, frequency stabilization food, etc.);
on methods of reducing the effect of external and internal influencing quantities (screening from magnetic and electric fields, depreciation, leveling, etc.).

If represent the error from the action of an external influencing quantity in the form of the product of the influence coefficient and the influencing quantity, then in design, in order to increase the accuracy, the influence coefficient is reduced, and using methods of increasing accuracy, the influencing quantity itself or its change is reduced (without changing the influence coefficient).



Accuracy Improvement Methods Based on Reducing the Existing Error

Statistical minimization methods are used mainly to reduce the random component of the error. The main statistical minimization is the averaging of the results of measurements or transformations that contain random, independent errors. Time averaging is used based on multiple measurements carried out at time intervals exceeding the correlation interval, and spatial averaging, which is implemented by several SI with independent random errors.

Correction methods can be used to reduce both the systematic and random components of the error.

Depending on the way of implementation, there are two main methods of correction:

- using the transformation of external actions or uninformative parameters that cause an error into other physical quantities in order to use them to improve the accuracy of the measurement result or measurement conversion;
- with the identification of an error that has arisen with the help of exemplary or redundant SIT for its further use in order to improve the accuracy of the measurement result or measuring conversion.



Methods for increasing the accuracy,

which are aimed at reducing the error from the action of influencing quantities, can be divided into:

- **Invariant** - when using invariant methods, SI is invariant to the corresponding influencing quantity, i.e. the error is zero.
- **Innocent** - when using innocent methods of increasing the accuracy, errors of the second and higher orders remain.

- **Minimizing error** - when minimizing, the error is reduced to the required level.

If we represent the error from the effect of the influencing quantity in the form

$$\Delta = \Psi(\xi - \xi_{\text{ref}})$$

Where $\Psi(\xi)$ is the influence function;

ξ - influencing quantity;

ξ_{ref} is the normal value of the influencing quantity.

In invariant methods

$$\Psi(\xi - \xi_{\text{ref}}) = 0$$

If we represent the influence function as a power polynomial

$$\Delta = a(\xi - \xi_{\text{ref}}) + b(\xi - \xi_{\text{ref}})^2 + c(\xi - \xi_{\text{ref}})^3 + \dots$$

Then in the innocent methods the components remain

$$\Delta = b(\xi - \xi_{\text{ref}})^2 + c(\xi - \xi_{\text{ref}})^3 + \dots$$




Methods for correcting the systematic component of the error

Permanent bias correction methods

Correction methods usually use redundancy, i.e. besides one main measurement, one more is needed to correct the measurement result.

Correction methods can use substitution, sign reversal (inversion) of a systematic error or input value, and correction.

Systematic error can be eliminated using the substitution method if the experimenter has at his disposal an adjustable measure, the output value of which is homogeneous with the input value of the measuring instrument.



The method of changing the sign of the systematic error is implemented if the sign of the error Δ_s can be changed while maintaining the sign of the measured value x . In this case, the error from the influencing quantity is compensated.

The measurements are carried out in two stages, while in the second stage with the opposite direction of the influencing quantity. In this case, with a change in the sign of the influencing quantity, the sign of the systematic error should also change.

The measurement result at the first stage is $x' = x - \Delta_s$ (1)

The measurement result at the second stage after changing the sign of the influencing quantity or the direction of its action is

$$x'' = x + \Delta_s \quad (2)$$

The measurement result with the exclusion of systematic error is obtained as

$$x = \frac{1}{2}(x' + x'') \quad (3)$$



Example 1

Thus, the error from the action of external magnetic fields is eliminated. In an astatic device (i.e. in a device that does not depend on the spatial position), the measuring mechanism consists of two identical parts, which makes it possible to automate the above steps to eliminate systematic errors.

The external magnetic field acts in the opposite way on each of the identical parts of this mechanism, i.e. equations (1) and (2) are realized.

Combining these parts in one mechanism makes it possible to implement equation (3), i.e. obtaining a corrected measurement result.



Example 1

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Example 2

The method of inverting the input value is based on the possibility of changing the sign of the measured value while maintaining the sign and systematic error.

First measurement result

$$x' = x + \Delta_s$$

Reversing the sign of x , we obtain

$$-x'' = -x + \Delta_s$$

Then the corrected measurement result

$$x = \frac{1}{2} (x' - x'')$$



Variable bias correction methods

To eliminate the variable systematic error, correction tables (if the error model is known), the method of symmetrical measurements, the method of periodic measurements.

To eliminate systematic progressive error, the method of symmetric measurements is used. Let us consider an example of eliminating the systematic multiplicative error of a measuring device, which varies linearly with time. This error can be eliminated by using a measure that reproduces x_0 and a measurement that consists of three stages. Separate measurements (stages) are performed after a certain period of time T . At the output of the device we obtain:

$$x_1 = x_0 + \Delta_{s0} \quad (\text{First stage});$$

$$x_2 = x_0 + \frac{\partial \delta_s}{\partial t} x_0 T + \Delta_{s0} \quad (\text{Second stage})$$

$$x_3 = x_0 + \frac{\partial \delta_s}{\partial t} x_0 2T + \Delta_{s0} \quad (\text{Third stage})$$



Where: Δ_{s0} is the initial value of the error;

$\frac{\partial \delta_s}{\partial t}$ - rate of change of the systematic component of the error.

In this case, the progressive multiplicative error is excluded when calculating x by the formula

$$x = \frac{x_2 - x_1 + x_0}{2x_0 + x_3 - x_1} \cdot 2x_0$$

If the systematic error changes periodically, then to eliminate it (when measuring a constant value), the method of periodic measurements can be used. Then, to eliminate this component of the error, two measurements are made at a time interval that is equal to half the period of change of the systematic error. Then, in two dimensions, systematic errors are equal in value and opposite in sign. To calculate the result, find the average and eliminate the error.



Statistical minimization method

In statistical minimization, the results of multiple measurements are processed in accordance with the selected algorithm. The measurement result is obtained from the condition of the minimum error.

For most distributions of random errors this condition is met by the expectation $M[x]$ and its estimate is the arithmetic mean

$$\bar{x} = \tilde{M}[x] = \frac{1}{n} \sum_{i=1}^n x_i$$


In the absence of a systematic error and a constant value of the measured value, the measurement result

$$\bar{x} = x_{\text{true}} + \dot{\Delta}_p$$

Where $\dot{\Delta}_p = \frac{1}{n} \sum_{i=1}^n \dot{\Delta}_i$ is the random error of the measurement result.

Arithmetic mean is a consistent and unbiased estimate of the true value of the measured quantity. Those. in the absence of a systematic error, the arithmetic mean is the value, the mathematical expectation of which is the true value, and the dispersion is characterized by the standard deviation $\sigma[\bar{x}]$

The arithmetic mean (measurement result) is a random variable "in small", i.e. with a small amount of randomness, the bulk of the average is a non-random value.



The characteristic of the accuracy of the obtained result \bar{x} is its standard deviation $\sigma[\bar{x}]$. Let us find $\sigma[\bar{x}]$ for the case when the general standard deviation of the measurement results $\sigma[x]$ is known, and the measurement results are independent

$$D[\bar{x}] = D\left[\frac{1}{n} \sum_{i=1}^n x_i\right] = \frac{1}{n^2} \sum_{i=1}^n D[x] = \frac{n\sigma^2[\dot{\Delta}]}{n^2} = \frac{\sigma^2[\dot{\Delta}]}{n}$$

$$\sigma[\bar{x}] = \sigma[\dot{\Delta}_p] = \frac{\sigma[\dot{\Delta}]}{\sqrt{n}}$$

Thus, when determining the measurement result as the arithmetic mean of n measurements, the standard deviation of the measurement result error decreases by \sqrt{n} factor of comparison with the general standard deviation of the results of individual measurements.



Conclusion:

In this lecture, you got acquainted with the main methods of increasing the accuracy of measurements.

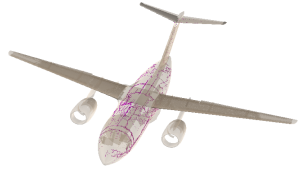
But to correct systematic errors, it is necessary to conduct a preliminary study to detect them and identify the type of systematic error.

The next lecture is devoted to these questions.



**Thank you for your
attention!**





The course
theme:

Methods and means of measurement

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Lecture 8
**METHODS FOR
DETECTING SYSTEMATIC
ERRORS**



The purpose of the lecture is to study ways to detect systematic error.

For ease of study, the methods for detecting systematic errors are classified, depending on the nature of the systematic error, into methods for detecting constant and variable systematic errors.

It is advisable to reduce or compensate the systematic component of the measurement error by lowering it to the level of the random component of the error.

If the systematic and random components of the error are comparable, then the first can be detected only by statistical methods.

In this case, the methods for correcting the systematic error are based on statistical methods for its detection.


The inequalities (criteria) given in the lecture make it possible to detect the systematic component of the error if it exceeds the random component of the error averaged by statistical methods.



Methods for detecting variable systematic errors

Variable systematic errors can be found in a number of multiple measurements of the same quantity.

Methods for detecting variable errors are divided according to the significance of the random component of the error.



1.1. Methods for detecting variable systematic errors with an insignificant random component of the error

If the random error is insignificant, the systematic component of the error can be detected by the alternation of signs of random deviations from the arithmetic mean.

The rules for detecting variable systematic errors in multiple measurements are given below:

- If the signs of uncorrected random deviations alternate with a certain pattern, then there is a variable systematic error.
- If the sequence of signs + random deviations is replaced by a sequence of signs - and vice versa, then there is a progressive systematic error.
- If the groups of signs + and - of random deviations alternate, then there is a periodic systematic error.



1.2. Methods for detecting variable systematic errors with a significant random component of the error

Provided the rules for detecting systematic errors can be used if the random component of the error is insignificant.

If the random component of the error is significant, then the criteria for checking the independence of the sample values and the stationarity of the sample are used to detect the systematic component of the error.

It is a run test based on the median and a downward and upward run test.

1.2.1. Criterion of set, based on median

When using the criteria of test, based on the median, a set of measurements are presented as a set in ascending order of results. The average element of the variation set is taken as the sample median value, i.e.

$\tilde{x}_{med} = X_{(\frac{n+1}{2})}$ if n is odd, $\tilde{x}_{med} = \frac{1}{2} X_{(\frac{n}{2})} + X_{(\frac{n}{2}+1)}$, if n is even. Then return to the initial sample (without ranking), and instead of each x_i they put "+", if $x_i > \tilde{x}_{med}$ and "-" if $x_i < \tilde{x}_{med}$ (members of the sample that are equal \tilde{x}_{med} in the sequence of pluses and minuses thus obtained are skipped). The resulting sequence of pluses and minuses is characterized by the total number of sets $v(n)$ and the length of the longest set $\tau(n)$.

A "set" is understood as a sequential series of pluses or minuses (the shortest series consists of one "+" and one "-"). For the level of significance $0,05 > \alpha > 0,0975$, provided that at least one of the inequalities is violated, the hypothesis about the statistical independence of random deviations (or measurement results) is discarded and the hypothesis about the presence of a systematic component of the error is accepted.

$$v(n) > [\tfrac{1}{2}(n+1) - 1,96\sqrt{n-1}],$$

$$\tau(n) < [3,3\lg(n+1)]$$



1.2.2. The criterion of "ascending" and "descending" series

The criterion of "ascending" and "descending" series is sensitive to progressive and periodic systematic errors. It also investigates the sequence of signs - pluses and minuses, but the rule for obtaining this sequence is as follows. If the sample has the form x_1, x_2, \dots, x_n at the i -th place of this sequence, put "+", if $x_{i+1} - x_i > 0$, and minus if $x_{i+1} - x_i < 0$ (if they are equal to each other, then only one of them is taken into account). For the significance level, $0,05 < \alpha < 0,0975$ this criterion has the form

$$v(n) > \left[\frac{1}{3}(2n + 1) - 1,96 \sqrt{\frac{16n - 29}{90}} \right],$$
$$\tau(n) < \tau_0(n),$$

Where $\tau_0(n)$ depending on n is equal to:

$$\begin{aligned} n \leq 26; \tau_0(n) &= 5; \\ 26 < n \leq 153; \tau_0(n) &= 6; \\ 153 < n \leq 1170; \tau_0(n) &= 7. \end{aligned}$$

If at least one of the inequalities is violated, then the hypothesis of the statistical independence of random deviations or measurement results is discarded and the hypothesis of the presence of a systematic component of the error is accepted. To detect progressive systematic errors in a normally distributed population, the Abbe test is used.

2. Methods for detecting permanent systematic errors

With an insignificant random component of the error, a constant systematic error can be detected with a single measurement using an exemplary measuring tool. With a significant random component of the error, a constant error can be detected in the presence of multiple measurements of the same value, performed by the measuring tool with a systematic error and an exemplary measuring tool.

If each measurement result x_i includes a systematic random component of the error $x_i = x_{\text{HTT}} + \Delta_s + \Delta_i$ then the arithmetic mean of a series of measurements includes a systematic and averaged random error $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = x_{\text{HTT}} + \Delta_s + \frac{1}{n} \sum_{i=1}^n \Delta_i$

Random errors in the composition \bar{x} have different signs and therefore their average value is much less than the components and is close to zero for a large number of measurements, but the constant systematic component of the error remains unchanged. Therefore, it \bar{x} is called the uncorrected mean. To correct it, it is necessary, if possible, to detect and correct the systematic component of the error. If the systematic error is constant, then U_i - random deviations of the measurement results from the arithmetic mean, do not depend on it, i.e. $U_i = x_i - \bar{x} = \Delta_i - \frac{1}{n} \sum_{i=1}^n \Delta_i$ therefore, uncorrected random deviations can be directly used to estimate the dispersion of a series of measurements.

But from one uncorrected series of measurements, it is impossible to find a constant systematic component of the error.

Therefore, the second row of measurements is obtained either with the help of an exemplary measuring tool or with the help of measuring tool, in which there is practically no systematic component of the error. If it is known that the results of the two groups are normally distributed, the mean values \bar{x}_1 and \bar{x}_2 , the general standard deviation $\sigma[x_1]$ and $\sigma[x_2]$ then the Laplace criterion is used to detect the systematic component of the error

$$z' = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\sigma^2[\bar{x}_1] + \sigma^2[\bar{x}_2]}} \quad \text{Where} \quad \sigma[\bar{x}_1] = \frac{\sigma[x_1]}{\sqrt{n_1}}, \quad \sigma[\bar{x}_2] = \frac{\sigma[x_2]}{\sqrt{n_2}}, \quad n_1, n_2 - \text{the number of}$$

measurements in the group. To test the hypothesis of the absence of a systematic error, the significance level is set α and the relative quantile is determined $z_{1-\frac{\alpha}{2}}$, which corresponds to the confidence level $P_{\text{доп}} = 1 - \alpha$. If $z' > z_{1-\frac{\alpha}{2}}$, then $z_{0,25} = 1,96$, then at $z' > 1,96$ it can be argued that there is a systematic error. If the general standard deviations are unknown, then the Student's t test is used to estimate the arithmetic mean

$$t' = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{(n_1 - 1)s^2[x_1] + (n_2 - 1)s^2[x_2]}} \cdot \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{n_1 + n_2}}$$

Where $s[x_1]$ and $s[x_2]$ - sample estimates of standard deviation that belong to the same general population:

If $t' > t_{1-\alpha/2}$, where $t_{1-\alpha/2}$ the quantile of the Student's distribution, found by the significance level $\alpha = 1 - P_{\text{доп}}$ and the number of degrees of freedom of the Student's distribution $n_1 + n_2 - 2$, then the hypothesis of the absence of a systematic error is discarded with the significance level α . To determine the quantiles of the Student's distribution, [Table 1](#) can be used. If the standard deviation of measurement groups does not belong to the same general population, then the coefficient t is

$$t' = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{s^2[x_1]/n_1 + s^2[x_2]/n_2}}$$

Systematic errors are present in one of the groups, if $t' > |t(q, f)|$, where is the level of significance ($q = \alpha/2$), f is the number of degrees of freedom, which is an integer $f = E(f' + 1)$,

$$f' = \frac{(n_1 - 1)(n_2 - 1)(s^2[x_1]/n_1 + s^2[x_2]/n_2)^2}{(n_1 - 1)s^4[x_1]/n_1^2 + (n_2 - 1)s^4[x_2]/n_2^2}$$

The quantiles of the Student's distribution depending on the level of significance q and the number of degrees of freedom f are given in [Table 1](#).



Distribution of Student's test

$$P\{|t| < t_p\} = 2 \int_0^{t_p} p(t, n) dt$$

	P			
$n - 1$	0.90	0.95	0.98	0.99
1	6.314	12.706	31.821	63.657
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.571	3.365	4.032
6	1.943	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
16	1.746	2.120	2.583	2.921
17	1.740	2.110	2.567	2.898
18	1.734	2.101	2.552	2.888
19	1.729	2.093	2.539	2.861
20	1.725	2.086	2.528	2.845
21	1.721	2.080	2.518	2.831
22	1.717	2.074	2.508	2.819
23	1.714	2.069	2.500	2.807
24	1.711	2.064	2.492	2.797
25	1.708	2.060	2.485	2.787
26	1.706	2.056	2.479	2.779
27	1.703	2.052	2.473	2.771
28	1.701	2.048	2.467	2.763
29	1.699	2.045	2.462	2.756
30	1.697	2.042	2.457	2.750
	1.64485	1.95996	2.32634	2.57582



Conclusion:

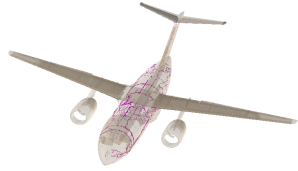
The methods you have studied for detecting a systematic error depend on its type (constant, variable) and on the ratio between the systematic and random components of the error.

The detected systematic errors can be further excluded or reduced.



**Thank you for your
attention!**





The course
theme:

Methods and means of measurement

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Lecture 9
**MEASUREMENT ERROR
MODEL. ACCURACY
CLASSES OF MEASURING
EQUIPMENT**



The purpose of the lecture is to study methods of standardizing accuracy classes of measuring instruments and methods of calculating measurement errors by designations of accuracy classes.

Lecture plan:

1. Selection of the error model

- 1.1. General information
- 1.2. Stationary error model
- 1.3. Non-stationary model of error

2. Standardization of measurement technology tools errors

- 2.1. Measurement technology tools accuracy classes
 - 2.1.1. Normalization of the accuracy class by relative error
 - 2.1.2. Standardization of the accuracy class according to the reduced error
 - 2.1.3. Standardization of the accuracy class with two components of the error
- 2.2. Designation of measurement technology tools accuracy classes



1. Selection of the error model

1.1 General information

Models of measurement errors and SIT should take into account the properties of errors in accordance with the above systematization. The model of the measurement error, taking into account the nature of the error change (random, systematic) and the nature of the dependence of the error on the measured value, is presented in the form of a polynomial formula. Since errors are considered as random variables or processes, the exhaustive characteristics of which are the probability distributions of their values, the distributions (density or distribution functions) can be unconditional, conditional, joint.

The polynomial model of the absolute error has the form:

$$\Delta(x) = a + b \cdot x + c \cdot x^2 + \dots = a_s + b_s \cdot (x) + c_s \cdot x^2 + \dots + \overset{\circ}{a} + \overset{\circ}{b} \cdot x + \overset{\circ}{c} \cdot x^2 + \dots$$

where a_s , b_s , $\overset{\circ}{a}$, $\overset{\circ}{b}$, $\overset{\circ}{c}$ - the components or factors of systematic and random errors, respectively. ***The polynomial model of the relative error has the form:***

$$\delta(x) = \frac{a}{x} + b + c \cdot x + \dots = \frac{a_s}{x} + b_s + c_s \cdot x + \dots + \frac{\overset{\circ}{a}}{x} + \overset{\circ}{b} + \overset{\circ}{c} \cdot x + \dots$$

If we designate the conditional distribution of the error $p(\Delta/x)$, then the **conditional mathematical expectation and conditional variance** will be equal, respectively:

$$M[\Delta(x)/x] = \int_{-\infty}^{\infty} \Delta(x) p(\Delta/x) d\Delta$$

$$\sigma^2[\Delta(x)/x] = \int_{-\infty}^{\infty} \{\Delta(x) - M[\Delta(x)/x]\}^2 p(\Delta/x) d\Delta = M[\Delta^2(x)/x] - M^2[\Delta(x)/x]$$

When standardizing the measurement technology tools errors, where possible, a **simplified model is adopted taking into account** two components of the error: **additive and multiplicative**.

$$\Delta(x) = a + b \cdot x = \Delta_a + \delta_M \cdot x = \Delta_a + \Delta_M$$

$$\delta(x) = \frac{a}{x} + b = \frac{\Delta_a}{x} + \delta_M = \delta_a + \delta_M$$

Depending components Δ_a , δ_a , Δ_M , δ_M and the total deviation, the measured values are given in Fig.1.

If consider (for simplicity) the systematic error as not changing over time, i.e. $a_s = \text{const}$, $b_s = \text{const}$, then the **conditional mathematical expectation** on for a certa b_s value x is $M[\Delta(x)/x] = a_s + b_s \cdot x = \Delta_{as} + \delta_{Ms} \cdot x$

and the **conditional variance** is determined only by the characteristics of random errors:

$$D[\Delta(x)/x] = \sigma^2[\Delta(x)/x] = M \begin{bmatrix} \overset{\circ}{a}^2 \end{bmatrix} + x^2 \cdot M \begin{bmatrix} \overset{\circ}{b}^2 \end{bmatrix} + 2 \cdot x \cdot M \begin{bmatrix} \overset{\circ}{a} \overset{\circ}{b} \end{bmatrix}$$

If you accept

$$M\left[\overset{\circ}{a}^2\right] = \sigma^2\left[\overset{\circ}{a}\right], \quad M\left[\overset{\circ}{b}^2\right] = \sigma^2\left[\overset{\circ}{b}\right], \quad M\left[\overset{\circ}{a}\overset{\circ}{b}\right] = \sigma\left[\overset{\circ}{a}\right] \cdot \sigma\left[\overset{\circ}{b}\right] \cdot r_{ab},$$

Where r_{ab} - the correlation coefficient; $\overset{\circ}{a}$, $\overset{\circ}{b}$, then

$$\sigma^2[\Delta(x)/x] = \sigma^2\left[\overset{\circ}{a}\right] + x^2 \cdot \sigma^2\left[\overset{\circ}{b}\right] + 2 \cdot x \cdot \sigma\left[\overset{\circ}{a}\right] \cdot \sigma\left[\overset{\circ}{b}\right] \cdot r_{ab}$$

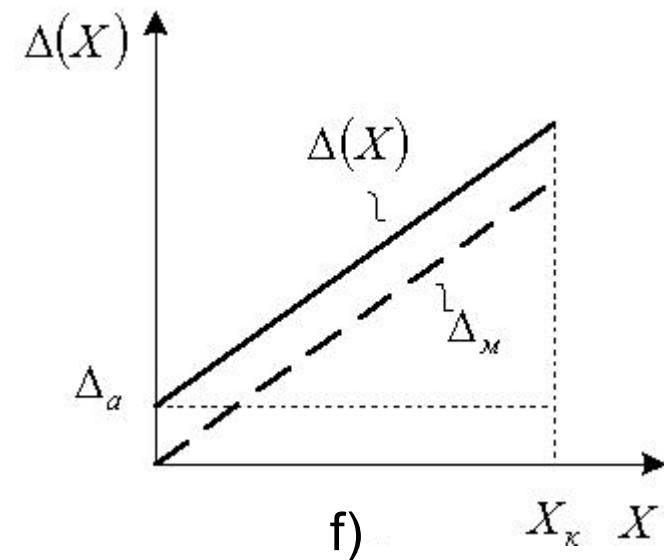
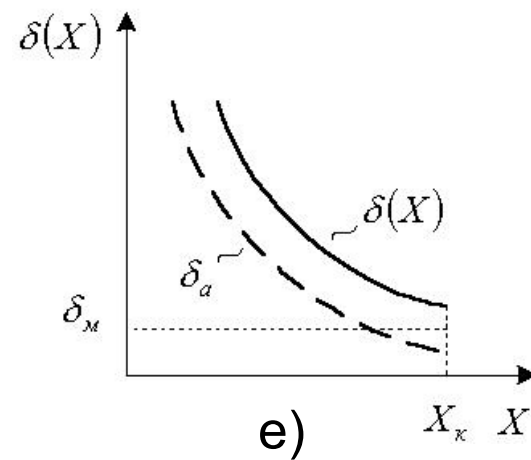
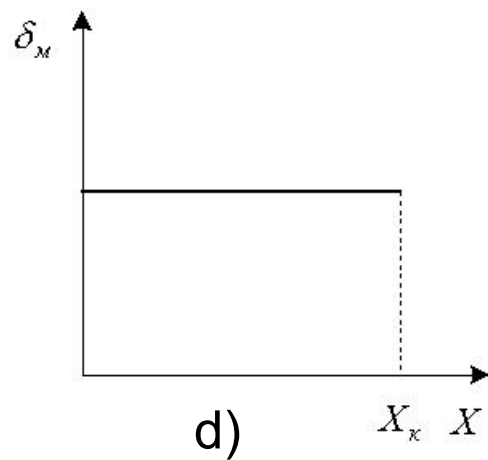
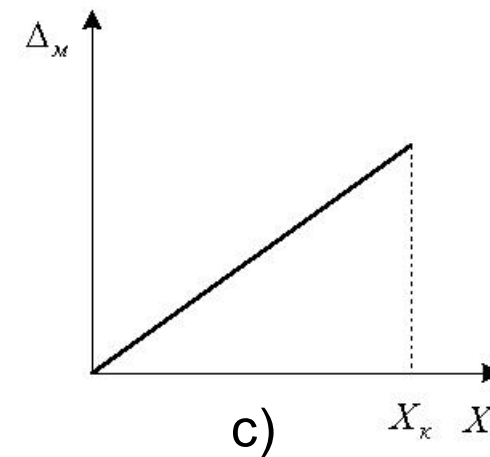
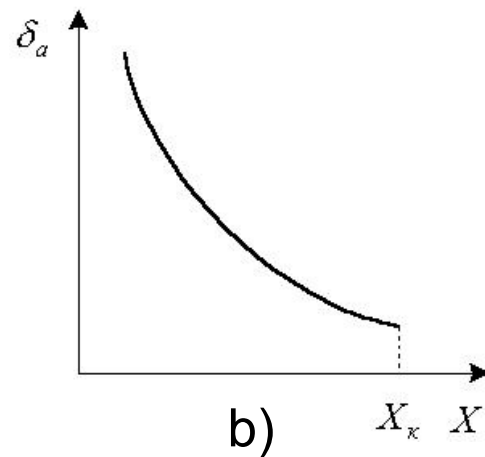
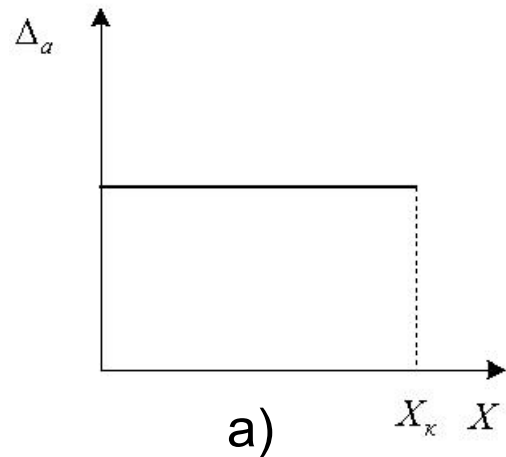
With independence of $\overset{\circ}{a}$ and $\overset{\circ}{b}$

$$\sigma^2[\Delta(x)/x] = \sigma^2\left[\overset{\circ}{a}\right] + x^2 \cdot \sigma^2\left[\overset{\circ}{b}\right] = \sigma^2\left[\overset{\circ}{\Delta}_a\right] + x^2 \cdot \sigma^2\left[\overset{\circ}{\delta}_M\right]$$

The absolute additive error does not depend on the measured value and can be characterized by an unconditional distribution. The same can be said for the relative multiplicative error. Therefore, **when standardizing the characteristics of the measurement technology tools errors, two components of the error are used separately - the absolute additive and the relative multiplicative.** If the measurement technology tools error is considered over a long period of time, then to solve many metrological problems, a **stationary or non-stationary model of a** random error is used.



Fig.1. Graphs of absolute (a) and relative (b) additive error; absolute (c) and relative (d) multiplicative error; absolute (e) and relative (f) total error.



1.2. Stationary error model

The first model (stationary random function) is the simplest, because it simulates an error if its non-stationarity can be neglected.

Such a model assumes a constant systematic component of the error Δ_s and a random component of the error, which is a stationary centered quantity $\dot{\Delta}(t)$

Then the measurement error Δ is characterized by **mathematical expectation $M[\Delta]$ and standard deviation $\sigma[\Delta]$** .

$$M[\Delta] = \Delta_s ; \quad \sigma[\Delta] = \sigma[\dot{\Delta}] .$$

In order to increase the accuracy, the systematic component of the error can be eliminated. But due to its inaccurate estimation, there remains a non-excluded part of the systematic error, which can be characterized by the boundaries of the interval with a given probability or standard deviation $\sigma[\Delta_s]$. In the latter case, the **standard deviation of the measurement error is**

$$\sigma[\Delta] = \sqrt{\sigma^2[\dot{\Delta}] + \sigma^2[\Delta_s]}$$

but the mathematical expectation $M[\Delta] = 0$.

1.3. Non-stationary error model

As a non-stationary model of the error, a function can be used, which is the sum of a stationary centered random variable and a linear function of time

$$\Delta(t) = \Delta_s + b_s t + \overset{\circ}{\Delta}(t)$$

The model reflects the influence of such factors as aging, linear drift, which cause a monotonic, close to linear change in error over time. If the error is analyzed over a long period of time, then it is presented as the sum of two stationary random functions of time: “low-frequency” $\overset{\circ}{\Delta}_H(t)$ and “high-frequency” $\overset{\circ}{\Delta}_B(t)$.

$$\Delta(t) = \Delta_s + \overset{\circ}{\Delta}_H(t) + \overset{\circ}{\Delta}_B(t)$$

In this case, the error characteristics are supplemented with normalized autocorrelation functions or power spectral densities.

Based on the frequency spectrum of the error, the low-frequency component is considered to be one whose autocorrelation function $\overset{\circ}{\Delta}_B(t)$ decays in a time interval that is less than the time of one measurement observation.



2. Standardization of measurement technology tools errors


2.1. Measurement technology tools accuracy classes

If the measurement technology tools errors are standardized without dividing into systematic and random, the accuracy class is used to normalize the characteristics of the measurement technology tools errors.

Accuracy class of measurement technology tools - generalized characteristic of measurement technology tools, which is defined by the boundaries of the basic and additional error.

The accuracy class of the measurement technology tools, although it characterizes the properties of the measurement technology tools with respect to accuracy, is not a direct indicator of the accuracy of measurements that are carried out using the measurement technology tools.

- Measurement technology tools with two or more ranges of the same value may be assigned two or more accuracy classes.
- In order to limit the nomenclature of measurement technology tools in terms of accuracy, a limited number of accuracy classes are established for a specific measurement technology tools type.

- 
- Accuracy classes of measurement technology tools system with computing components can be set without a computing component. Then the errors of the computational component are normalized separately.
 - If several accuracy classes are set for measurement technology tools, then, if necessary, you can set one, but the one that corresponds to the greatest error. For example, the accuracy class of a set of measures is determined by the accuracy class of the measure with the greatest error.
 - Measurement technology tools, which are intended to measure different physical quantities, may have different accuracy classes for individual quantities.
 - The boundaries of the basic and additional errors of the measurement technology tools (change in readings) of a certain accuracy class are set in the form of absolute, relative or reduced errors, depending on the nature of their relationship with the measured value (Fig. 1).

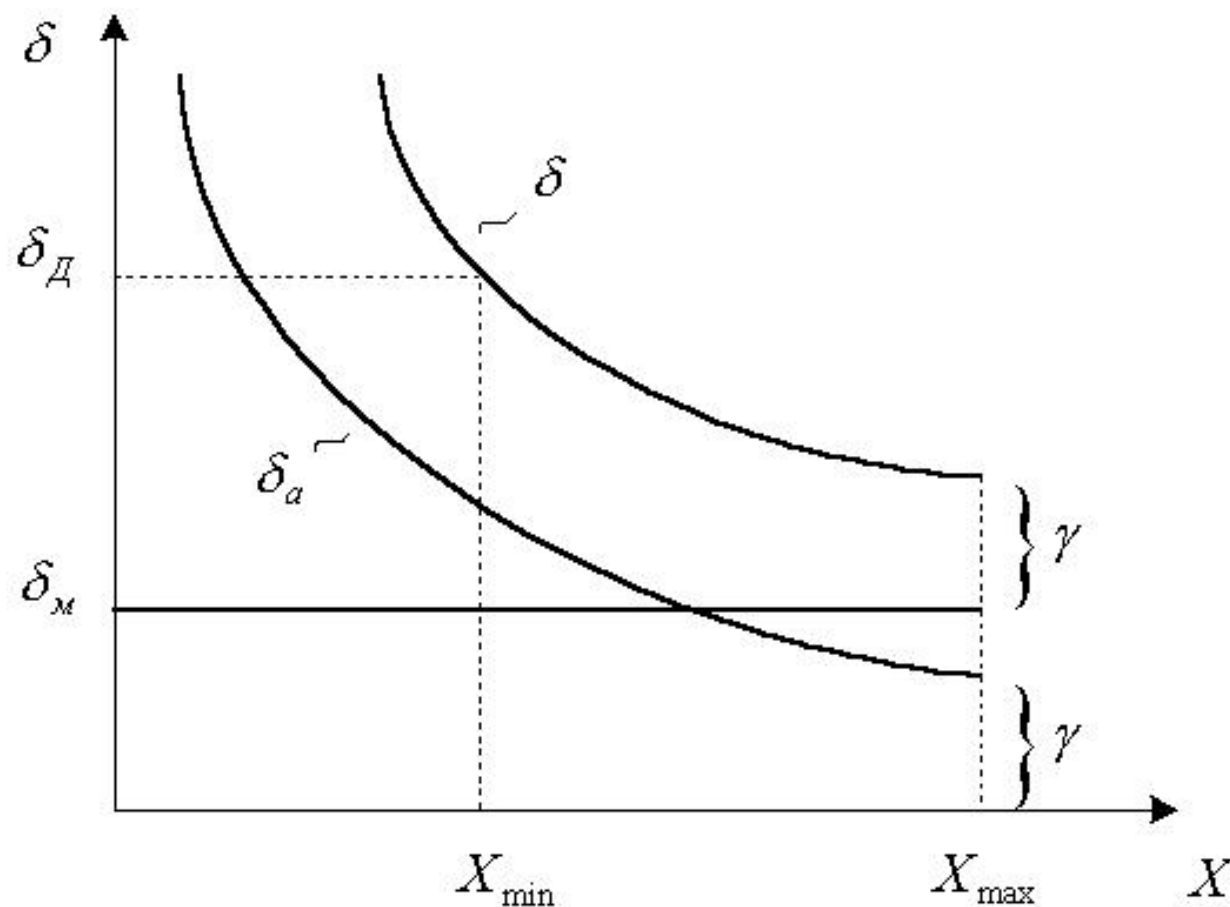
2.1.1. Standardization of the accuracy class by relative error

If the error is purely multiplicative, then it, as an absolute, linearly depends on the measured value. Therefore, the relative multiplicative error is constant and does not depend on the measured value. This is used when standardizing the accuracy class. In this case, the accuracy class is the boundary of the permissible relative error in percent.

Thus, the errors of scale converters (voltage dividers, shunts, measuring current and voltage transformers, etc.) are normalized. With this normalization, the relative error is the same over the entire range. But it is difficult to create a converter that is completely free of additive errors. Errors due to noise, drift, friction, interference, etc. are inherent in many measurement technology tools. Therefore, the relative error is normalized by one number only in a limited range of the measured value. This excludes the initial part of the range (Fig. 2).

The reduced additive error γ_a is usually significantly less than the relative multiplicative error δ_M . But the total error, which is equal $\delta = \delta_a + \delta_M$, increases with decreasing x . If the permissible error δ_D is normalized, then the lower limit of the range x_{\min} is the abscissa, which has an ordinate $\delta_D = \delta$. Therefore, for real measurement technology tools whose error is normalized by one number - the relative error δ_D , always point measurements of the range x_{\min}, x_{\max} in which $\delta \leq \delta_D$. Thus, **the measurement range is the range of values of a quantity within which measurements are performed with a standardized error**

Fig. 2 Exclusion of the initial part of the range in the presence of a small additive error and normalization of the relative error as an accuracy class



2.1.2. Normalization of the accuracy class according to the reduced error


If the error is purely additive, then it, as an absolute one, does not depend on the measured value. But it is inconvenient to normalize the absolute error, since it is expressed in units of the measured value. Therefore, the reduced error is normalized γ , which is the ratio of the absolute error to the normalized value x_H .

If given γ , then the absolute and relative measurement errors are found as follows

$$\Delta = \gamma \cdot x_H, \quad \delta = \gamma \frac{x_H}{x}.$$

Then the accuracy class is the minimum measurement technology tools error, which is equal to the relative error at $x = x_H$. For the remaining values $x < x_H$, which is equal to the maximum value of the range, when $x = x_H/2$, $\delta = 2 \cdot \gamma$ and if $x = x_H/10$, $\delta = 10 \cdot \gamma$. With a further decrease x , the error δ approaches infinity. In connection with this dependence of the relative error on the measured value, the concept of a sensitivity threshold is introduced.

The sensitivity threshold $x_{\text{пч}}$ is the x value for which $\delta = 1 = 100$,% . Then the limit of the full scale or range of indications is $x_{\text{пч}}$ (start value) and x_{max} (end value).



The range of indications of a measuring instrument is the range of values of the measured quantity, which is limited by its initial and final values.


This is the measurement interval - the algebraic difference between the upper and lower limits of the measurement range).

If the threshold of sensitivity is neglected, then the limits of the range of indications $[0; x_{\max}]$.

The measurement range is usually smaller and has boundaries x_{\min} (low) and x_{\max} (high) where the beginning of the range x_{\min} meets $\delta \leq \delta_{\mathcal{A}}$.

The value $\delta_{\mathcal{A}}$ is set in accordance with the specific measurement task.

To determine the measurement error by the accuracy class, it is necessary to know the accuracy class and the normalizing value .



Normalizing value for measurement instrument with a uniform, practically uniform or power-law scale, as well as for transducers, if the zero value of the input (output) signal is at the edge or outside the range (indications), **set equal to or equal to the greater of the range limits** (indications) **from the modules of the range limits** (indications), **if the zero value is within the range** (indications).

- **For electrical measuring instruments with a uniform, practically uniform or power-law scale and a zero mark within the range of indications** , the normalizing value may be set equal to the **sum of the modules of the limits of the range of indications**.
- **For measuring instrument, which adopted a scale with a conditional zero**, the normalizing value is set equal to the modulus of the difference between the boundaries of the indication range.
- **For measuring instruments with established nominal values**, the standardizing value is set equal to this nominal value.
- **For measuring instruments with a substantially uneven scale**, the standardizing value is set equal to the **entire length of the scale or its part corresponding to the measurement range** . In this case, the limits of absolute error are expressed, like the length of the scale, in units of length.

2.1.3. Normalization of the accuracy class with two components of the error

In the presence of two components of the error (additive and multiplicative), the **absolute and relative total error is characterized by the ratios** $\Delta = \Delta_a + \Delta_M = \Delta_a + \delta_M \cdot x$, $\delta = \delta_M + \delta_a = \delta_M + \gamma \cdot \frac{x_H}{x}$

The **relative error** is represented as
$$\delta = \pm \left[c + d \left(\frac{x_H}{x} - 1 \right) \right], \%$$

where **c, d** are numbers that correspond to the standard series and which are obtained by rounding up from the error at the end of the range $\delta_M + \gamma$ and the reduced additive error γ respectively.

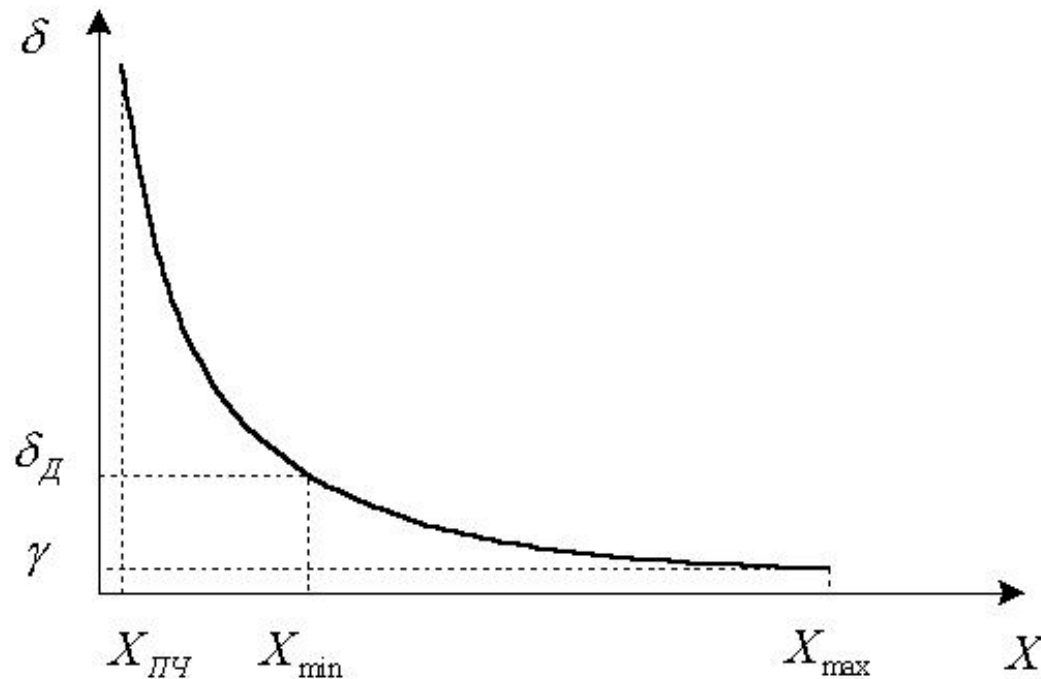
The standard series of numbers that is used to establish accuracy classes is:

1	1.5	(1.6)	2	2.5	(3)	4	5	(6)	10n
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n = 1; 0; -1 ... Values in parentheses are not recommended for development tools.

In the documentation, series of numbers are indicated for the basic requirements for various types and types of measurement technology tools to establish the accuracy classes of measurement technology tools data. For example, direct-acting analogue electrical measuring instruments and auxiliary parts to them, according to standards, the following series of numbers is established for assigning accuracy classes: 0.15; 0.2; 0.5; 1; 1.5; 2; 2.5; 3; 5.

For measurement technology tools, the boundaries of the permissible basic error of which are presented in the form of absolute errors $\Delta = \pm a$, $\Delta = \pm(a + bx)$, or relative errors, which are presented in the form of a graph or a formula that does not coincide with, the accuracy classes are denoted by letters of the Latin alphabet or Roman numerals.



Formation of the range of indications [$x_{\text{пч}}$, x_{\min}] and the range of measurements [x_{\min} , x_{\max}] in the presence of an additive error and normalization of the reduced error.




2.2. Designation of measurement technology tools accuracy classes

The designations of the accuracy classes of the measurement technology tools in the normative technical documentation and on the measurement technology tools are given in Table 1.

1. **The limits of permissible additional** (changes in readings) **errors** are established by:

- as a constant value for the entire working area of the influencing quantity or as constant values over the intervals of the working area of the influencing quantity;
- by indicating the ratio of the limit of the permissible additional error corresponding to the regulated interval of the influencing quantity to this interval;
- by indicating the dependence of the limit of the permissible additional error on the influencing quantity (limiting influence function);
- by indicating the functional dependence of the limits of permissible deviations from the nominal influence function.



2. **The limits of the permissible additional error** (change in readings), as a rule, are set in the form of a fractional (multiple) value of the limit of the basic permissible error.

3. **The limits of permissible variation of the measuring instrument readings** or the output signal of the transducer are set as a fractional (multiple) value of the basic permissible error limit.

4. The limits of permissible instability are also set in the form of a fraction of the basic permissible error.









In addition to the above methods of standardizing errors, more complex ones are also allowed. For example, some bridges use a three-term error formula to measure resistance

$$\delta = \frac{x'_{\min}}{x} + \delta_M + \frac{x}{x'_{\max}}$$

Or in the technical documentation for wide-range devices, instead of the three-term formula, subranges of the value are given, in which the error does not exceed the permissible value. The example of such a rationing is given. The bridge error does not exceed 0.5% in the subrange (102104) Ohm; 1% at (51105) Ohm; 5% at (0.51106) Ohm; 10% at (0.22106) Ohm; 20% at (0.14106) Ohm. The three-term formula for such an error has the coefficients

$$\delta(x) = 100 \cdot \left[\frac{0.02}{x} + 5 \cdot 10^{-3} + \frac{x}{20 \cdot 10^6} \right], \%$$

Table 1. Designation of accuracy classes of measurement technology tools in documentation

Error presentation form	Limits of permissible basic error		Accuracy class designations	
	Formula	Examples of	In the documentation	On SIT in accordance with standards
Given if:- the normalizing value in units of the value at the input (output) of the measurement technology tools (except for those cases when the normalizing value corresponds to the length of the scale or interval);	$\gamma = \frac{\Delta(x)}{x_H} = \pm p$	$\gamma = \pm 1.0$ $\gamma = \pm 1.5$	$\gamma = \pm 1.0$ $\gamma = \pm 1.5$ Accuracy class 1 1.5	1 1.5
- when the normalizing value corresponds to the scale length;	$\gamma = \frac{\Delta(L)}{L} = \pm p$	$\gamma = \pm 0.5$ $\gamma = \pm 1.0$	Accuracy class 0.5 1.0	 
- when the normalizing value corresponds to the measurement interval (indication range)	$\gamma = \frac{\Delta(x)}{x_H} = \pm p$	$\gamma = \pm 1.0$ $\gamma = \pm 2.5$	Accuracy class  	 
Relative	$\delta = \frac{\Delta}{x} = \pm q$ $\delta = \pm \left[c + d \cdot \left(\left \frac{x_H}{x} \right - 1 \right) \right]$	$\delta = \pm 0.5$ $\delta = \pm \left[0.02 + 0.01 \cdot \left(\left \frac{x_H}{x} \right - 1 \right) \right]$	Accuracy class 0.5 Accuracy class 0.02 / 0.01	  0.02 / 0.01
Absolute	$\Delta = \pm a$ $\Delta = \pm(a + b \cdot x)$	$\Delta = \pm 0.5 O_M$ $\Delta = \pm(0.5 + 0.01 \cdot X) O_M$	Accuracy class M	M
Relative	Graph, table, formula for δ		Accuracy class C	C



Conclusion:

The purpose of each measurement is to obtain a value of a quantity with an uncertainty, one of which is the measurement error.

Having studied this section, you can calculate the error of the measuring instrument by the sign of the accuracy class on the measuring instrument or by the formula for the error in the regulatory and technical documentation for the measuring instrument.



**Thank you for your
attention!**





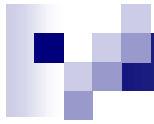
The course
theme:

Methods and means of measurement

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Lecture 10
**PRESENTATION OF THE
MEASUREMENT RESULT**



The purpose of this lecture is to explore ways of presenting a measurement result with an indication of uncertainty. This approach to the presentation of the result is proposed in the document of the International Organization for Standardization "Guide to the expression of uncertainty in measurement. ISO, 1993" and is currently used in most countries of the world.

Lecture plan:

1. The concept of uncertainty.
2. Forms of representation of uncertainty.
3. Methods for estimating uncertainty.
4. Presentation of the measurement result.
 - 4.1. Accepted designations.
 - 4.2. Significant numbers.
5. Examples of presentation of the result.



1. DEFINITION OF UNCERTAINTY

When the experimenter presents the result of his measurement, he indicates the best estimate of the measured quantity and the interval in which, as he is sure, most of the values of the measured quantity are.

The uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand. The result of a measurement after correction for recognized systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects.

The term uncertainty means doubt. Indeed, measurement uncertainty reflects doubt about the resulting estimate of the measurement.

The International Organization for Standardization document gives the following definition of measurement uncertainty:

Uncertainty (measurement) is a parameter combined with a measurement result that characterizes the dispersion of the measured value.



It is known that the parameter characterizing the scattering is the standard deviation or half the width of the scattering interval with the established confidence level.

The measurement error has two components: systematic and random. It is assumed that the measurement result can be corrected for all recognized systematic effects.

In practice, there are many possible sources of uncertainty in a measurement, including: a) incomplete definition of the measurand; b) imperfect realization of the definition of the measurand; c) nonrepresentative sampling — the sample measured may not represent the defined measurand; d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions; e) personal bias in reading analogue instruments; f) finite instrument resolution or discrimination threshold; g) inexact values of measurement standards and reference materials; h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm; i) approximations and assumptions incorporated in the measurement method and procedure; j) variations in repeated observations of the measurand under apparently identical conditions.



2. FORMS OF UNCERTAINTY REPRESENTATION

Standard uncertainty is the uncertainty of the measurement result, expressed as standard deviation.

The combined standard uncertainty is the standard uncertainty of a measurement result, which is used when the measurement result is obtained by measuring other quantities.

The combined standard uncertainty is equal to the positive root of the sum of the components (variances or covariances c^f the measured quantities), weighted according to the influence that the measured quantities have on the measurement result.

Expanded uncertainty - an uncertainty in the form of an interval around the measurement result, which contains a large part of the distribution of the values of the measured quantity.




3. METHODS FOR ESTIMATING UNCERTAINTY

The uncertainty of a measurement result usually consists of several components, which can be grouped into two categories according to the method from the assessment.

Components of category A are those that are statistically assessed. Components of category A are characterized by the dispersion S_i^2 (or standard deviation S_i) and the number of degrees of freedom ν_i , which depends on the number of measurements; covariance is given if necessary.

Components of category B are those that are assessed in a different way (using the rules of upper estimation, using a priori information). In this regard, any technique for assessing uncertainty must contain a report with a list of components and methods for their assessment. Components of category B are characterized by the values u_j^2 , which can be considered with approximation as corresponding to variances; the values u_j are considered similarly to standard deviation .

Measurement uncertainty has many components. Some of these components can be characterized by the statistical distribution of the results of a series of measurements and can be characterized by the standard deviation. These are components of category A. Other components can also be characterized by the standard deviation, but obtained on the basis of an accepted (from oneself) probability distribution using the experimenter's experience or other information. These components refer to category B.



The terms **confidence interval** or **confidence probability** are specific definitions of statistics and are used only when both the expanded uncertainty and the S_i components are obtained by type A estimates. Thus, in general, the word “confidence” does not refer to an interval. Taking into account the B type estimation, the expanded uncertainty is the interval, and P in this case is the covering probability or confidence level for the given interval.

The transition from standard to expanded uncertainty when estimating components of category B is carried out using a coverage factor (coefficient) k , which is selected based on the level of confidence required for the interval from that required for the interval from $x - U$ to $x + U$.

Usually k is within the range from 2 to 3.

However, for specific requirements, k can go beyond these limits.

In practice, you need to choose a coverage factor k that would provide the interval $x \pm k u(x)$ corresponding to the confidence level P , which is 95% or 99%.

Approximate recommendations: $k = 2$ for 95% and $k = 3$ for 99%.



4. PRESENTATION OF THE MEASUREMENT RESULT

4.1. Accepted designations

When recording the measurement result, the standard uncertainty is indicated by the letter u , the standard combined uncertainty by the letter u_c , and the expanded uncertainty by the letter U .

Examples:

1. x ; u ;

2. x , u_c ;

3. x U ; P ;

4. x U .

Example 4 is used for cases when $P = 1$.

In addition to the accepted types of uncertainty, when recording the result, a comment can be presented indicating the number of degrees of freedom (number of measurements), distribution, conditions of the experiment, etc.



4.2. Significant numbers


There are several basic rules for recording the result. First, since the uncertainty is an estimate of the error, it obviously cannot be given with very high accuracy.

It is incredible that a measurement error could be known to four significant figures. In the results of high-precision measurements, uncertainties with two significant figures are given, and for other measurements, one can be limited.

Thus, if some calculation of the uncertainty leads to the value $U = 0,04385 \text{ m/s}^2$, then this value should be rounded.

If the first digit in the uncertainty is 1, then it may be better to keep the two significant digits in the uncertainty.

For example, suppose that some calculation gives the uncertainty value $u = 0,14$. Rounding this value down to $u = 0,1$ means reducing the uncertainty by 40%; so it would be more correct to keep two digits and cast $u = 0,14$.



When the uncertainty in the measurement is calculated, it is necessary to analyze which numbers in the measured value are significant.

For example, a result of 92.81 with an uncertainty of 0.7 should be rounded to 92.8 0.7.

If the uncertainty is 7, then the result should be presented as 93 7.

This will reduce inaccuracies in rounding numbers.

At the end of the calculation, the final answer should be rounded and this additional digit should be removed.



5. EXAMPLES OF PRESENTATION OF THE RESULT.

Here are some examples of presenting the result.

- Example 1.

$m_s = 100.02147\text{g}$ with (combined standard uncertainty) $u_s = 0.35\text{ mg}$.

- Example 2.

$m_s = 100.02147\text{ (35) g}$, where the number in brackets is the numerical value (of the combined standard uncertainty) u_s , referred to the corresponding last digits of the result.

- Example 3.

$m_s = 100.02147\text{ (0.00035) g}$, where the number in parentheses is the numerical value (combined standard uncertainty) u_s , expressed in the units of the result.

- Example 4.

$m_s = (100.02147\text{ }0.00079)\text{ g}$, where the number following the symbol is the numerical value of (expanded uncertainty) determined from (combined standard uncertainty) $u_s = 0.35\text{ mg}$ and (coverage factor) $k = 2.26$, which is based on the t-distribution for $n = 9$ degrees of freedom, and represents the interval estimated for a confidence level of 0.95.



Consider a few additional considerations for presenting a measurement result.

The measurement result can be accompanied by a commentary, which provides information about the number of measurements, measurement conditions, etc.

If there are no special requirements for the choice of the confidence level, use $P = 0.95$.

For truncated uncertainty distributions, when recording the result, the boundary value can be used, which corresponds to the probability $P = 1$. This value of the level of confidence, as a rule, is not indicated. Then the measurement result is represented as

$x_i \pm U(x_i)$,

where $U(x_i)$ is the expanded uncertainty corresponding to probability 1.

■ Example 5.

$R = (1500 \pm 12) \text{ Ohm}$.

$I = (1.15 \pm 0.07) \text{ A}$.



Conclusion:

Thus, you have studied the forms of presentation of the measurement result with the help of uncertainty, which will later be used when recording the final result for single and multiple direct and indirect measurements.



**Thank you for your
attention!**






The course
theme:

Methods and means of measurement

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Lecture 11
**DIRECT SINGLE
MEASUREMENTS OF
QUASIDETERMINAL
VALUES**



The purpose of this lecture is to study the components of the error of a direct single measurement and how to combine them to present the result. In direct single measurements, the measured value is measured once and the resulting value is used as the measurement result.

However, it is often the case that in direct single measurements, several repeated measurements are made. In this case, redundant measurements are used to protect against hardware failures, and the measurement result is obtained one measurement at a time.

Lecture plan:

1. Analysis of the components of the error.
2. Estimation of the uncertainty of a direct single measurement.
3. Identification of sources of error.
4. Elimination or accounting of systematic components of the error.
5. Presentation of the result of a direct single measurement with one component of the error.
6. Combining the components of the error and the presentation of the measurement result (with several components of the error).
7. An example of a technique for combining errors.

1. ANALYSIS OF MEASUREMENT ERROR COMPONENTS

When evaluating the uncertainty of the result, information about the components of the uncertainty is used, i.e. about individual measurement errors.

The error of a single measurement Δ includes two components: methodical Δ_m and instrumental Δ_{in} .

$$\Delta = \Delta_m * \Delta_{in} \quad (1)$$

The sign (*) is a sign of combining components.

The instrumental error is the sum of the error in the interaction of the measuring instrument (MI) with the object of measurement Δ_b and the error of MI Δ_{MI} .

$$\Delta_{in} = \Delta_e * \Delta_{CH} \quad (2)$$

The MI error in the most general case consists of the main systematic component Δ_{os} , the main random component Δ_o , the main variation of H_o , the sum of additional errors Δ_{ci} from the influence of influencing quantities and uninformative parameters of the input signal, and the dynamic error Δ_d .

$$\Delta_{CH} = \Delta_{os} * \Delta_o * H_o * \sum_{i=1}^n \Delta_{ci} * \Delta_d \quad (3)$$



2. EVALUATION OF UNCERTAINTY OF DIRECT SINGLE DIMENSION

In data processing methods for each category of uncertainty components A and B, two groups of operations are distinguished, respectively: formal mathematical methods (mainly statistical) and non-formalized, specific metrological operations.

Some problems of data processing during measurements cannot be formalized, therefore, for them it is impossible to indicate an unambiguous, strictly justified solution. They should be resolved by willful means, in agreement with specialists - albeit not quite strictly, but uniformly, and their decisions should be recorded in documents. It is these circumstances, and not at all the lack of manuals on mathematical statistics, that necessitate the development of recommendations and normative and technical documents on methods of data processing during measurements.

This applies to a large extent to direct single measurements. The main steps in assessing the uncertainty of direct single measurements are as follows:

- 2.1. Identification of sources of error.

- 2.2. Elimination or accounting of systematic components of the error.

- 2.3. Combining the components of the error and presenting the result with an indication of the uncertainty.



3. IDENTIFICATION OF SOURCES OF ERROR

In practice, there are many sources of measurement uncertainty, including:

- a) imperfection in the definition of the measured quantity;
- b) imperfection in the implementation of the definition of the measurand (imperfection of the measurement method);
- c) non-representative sample;
- d) incomplete information on the influence of environmental parameters on the measurement or imperfect measurement under the given environmental conditions;
- e) personal inaccuracies (beats) in the reading of analog instruments;
- f) limited resolution;
- g) imprecise values of standards and reference materials;
- h) imprecise values of constants and other parameters obtained from external sources and used in algorithms;
- i) approximation and uncertainties in the method and procedure of measurement;
- j) changes in repeated measurements under constant conditions.

These sources of uncertainty do not have to be independent.



4. EXCLUSION OR ACCOUNTING OF THE SYSTEMATIC COMPONENTS OF ERROR

When analyzing the components of the error, they are divided into systematic and random. However, the recipient of the measurement often needs to know the overall uncertainty, regardless of the cause. In addition, the uncertainty of the result is indicated for a situation when systematic errors are corrected, and their non-excluded residuals are included in the total uncertainty.

Example 1.

During measurements, a measured value of 12.2 V was obtained.


The error due to interaction is –0.3 V with an uncertainty of 0.1 V.

Then the corrected value of the measured value is

$$x = 11,9 \text{ V with an uncertainty } \pm 0,1 \text{ V}$$

or

$$x = (11,9 \pm 0,1) \text{ V}$$



In the example above, the systematic error is presented as its value with an indication of the estimation uncertainty.

If the systematic error is set in the form of boundaries: lower Δ_{sl} and upper Δ_{sh} , then the boundaries, in which the true value of the measured value is located, are respectively equal:

lower $x_l = x - \Delta_{sl}$;

top $x_h = x + \Delta_{sh}$.

The half-sum is chosen as the best value of the measured quantity:

$$x_{best} = \frac{1}{2} (x_l + x_h) = x + \frac{1}{2} (\Delta_{sl} + \Delta_{sh}),$$

and the half- difference is used as an estimate of the uncertainty.

$$U = \frac{1}{2} (x_h - x_l) = \frac{1}{2} (\Delta_{sh} - \Delta_{sl}).$$

Example 2.

During measurements, the value of the measured quantity was 12,2 V. the result with an indication of the uncertainty has the form:

$x = 12,4 \text{ V with an uncertainty } \pm 0,2 \text{ V}$

 or

$x = (12,4 \pm 0,2 \text{ V}$



5. PRESENTATION OF THE RESULTS OF DIRECT SINGLE MEASUREMENTS WITH ONE COMPONENT OF ERROR

If during the measurement there is only one component of the error with boundaries $\pm \Delta$, then the measurement result can be represented with uncertainty in the form

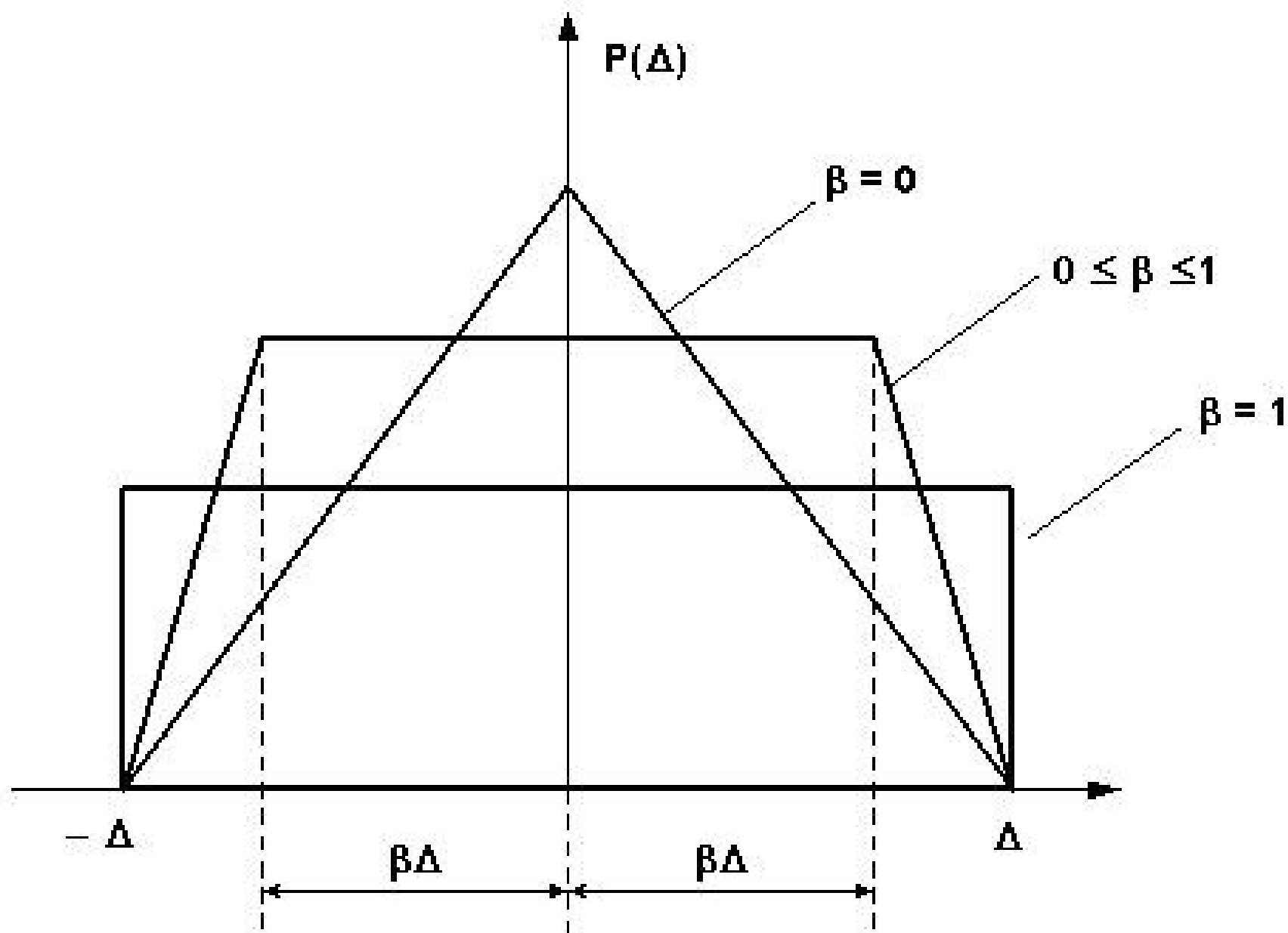
$$x \pm \Delta$$

or using standard uncertainty, i.e. standard deviation. To assess standard deviation, as a rule, assessment is used for components of category B.

When writing the result, use ***x*** and ***u*** .

The assignment of a uniform distribution provides an upper bound, but is not always physically justified. At the boundaries of the interval, the probability density is usually less. Therefore, when the probability of going beyond the boundaries of the interval is equal to 1, you can assign a triangular distribution or a trapezoidal distribution (Fig. 1).

Figure 1. Assignment of distributions for a given boundary Δ .



If uncertainty bounds $\pm \Delta$ and attributed trapezoidal distribution, the lower base of the trapezoid is 2Δ , and the upper $2 \beta \Delta$, wherein $0 \leq \beta \leq 1$. If $\beta = 1$, the trapezoidal distribution turns into rectangular, if $\beta = 0$, then - into triangular. If a trapezoidal distribution is assigned, then the variance $u^2 = \Delta^2 (1 + \beta^2) / 6$ and the standard deviation $u = \Delta \cdot \sqrt{1 + \beta^2} / \sqrt{6}$. For a triangular $u^2 = \Delta^2 / 6$, $u = \Delta / \sqrt{6}$. An illustration of the estimation of the standard uncertainty is shown in Fig. 2.

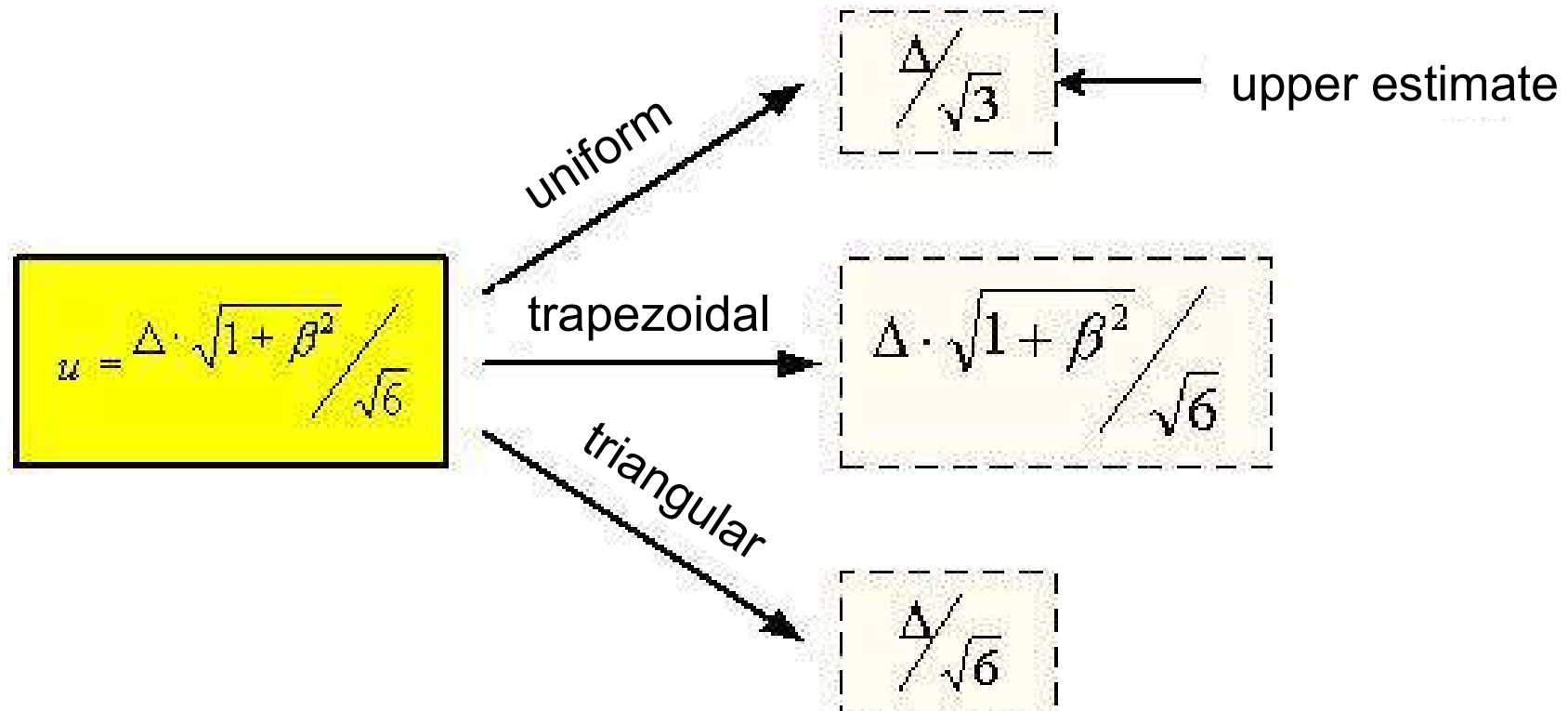


Figure 2. Estimation of standard uncertainty at given bounds.

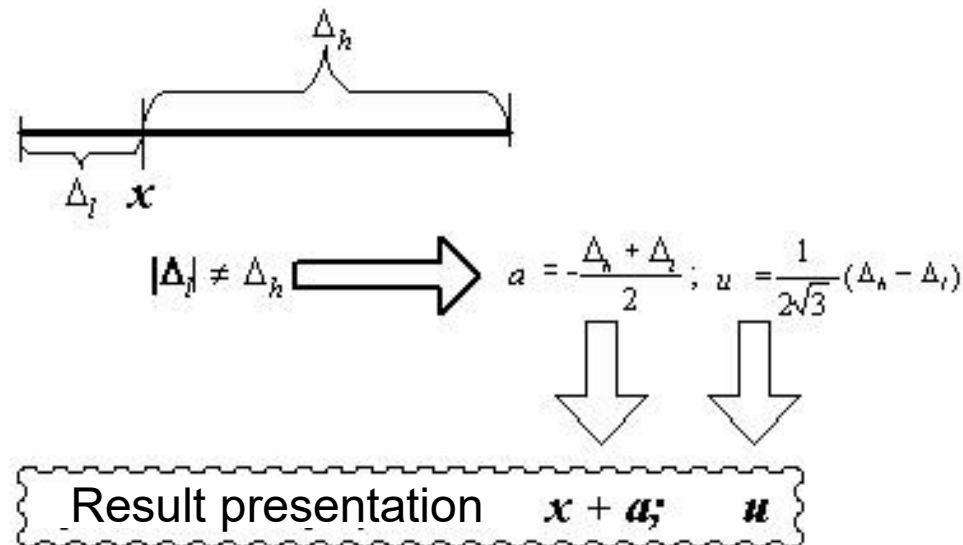
Example 3.

From the above examples of recording the result, it can be seen that the upper estimate corresponds to a uniform distribution of errors within the interval.

If the margin of error is asymmetrical, i.e. $x; \Delta_l; \Delta_h$, where Δ_l is the lower bound, and Δ_h is the upper bound, then the transition to recording the result with uncertainty has the form (Fig. 3.)

$$a = -\frac{\Delta_h + \Delta_l}{2}, \quad u^2 = \frac{(\Delta_h - \Delta_l)^2}{12},$$

where a is the correction, u^2 is the variance estimate. Then record the result $x + a; u$, where u is the standard uncertainty estimate.





6. COMBINATION OF ERROR COMPONENTS AND PRESENTATION OF THE MEASUREMENT RESULTS (WITH SEVERAL PARTS)

If there are several components of the error of a direct single measurement, then the problem arises of combining them. The individual components of the error can be represented as the boundary of the interval with a probability of 1, the boundary of the confidence interval, or standard deviation.

If the components of the error are presented as boundaries of an interval $\pm \Delta_i$ with probability 1, then the result with uncertainty in the form of an interval boundary with probability 1 can be represented as follows

$$x \pm \Delta, \quad (4)$$

Where $\Delta = \sum_{i=1}^n \Delta_i$

If the number of components is large, then the estimate Δ according to formula (4) will be overestimated, and in this case the expanded uncertainty is used in the form of the boundary of the interval $\Delta(P)$ for the confidence level P .

$$U = \Delta(P) = K(P) \cdot \sqrt{\sum_{i=1}^n \Delta_i^2}$$

where $K(P)$ is a coefficient that depends on the level of confidence P , the number of error components, and also on the ratio between the components (Fig. 4).

The values of the coefficients $K(P)$ for uniform distributions of components are given in **Table 1** (for uniform distributions of components). At $P = 0.90$, $P = 0.95$, the dependence of $K(P)$ on the number of components is insignificant, therefore it is recommended to take the average values $K(0.90) = 0.95$, $K(0.95) = 1.1$.

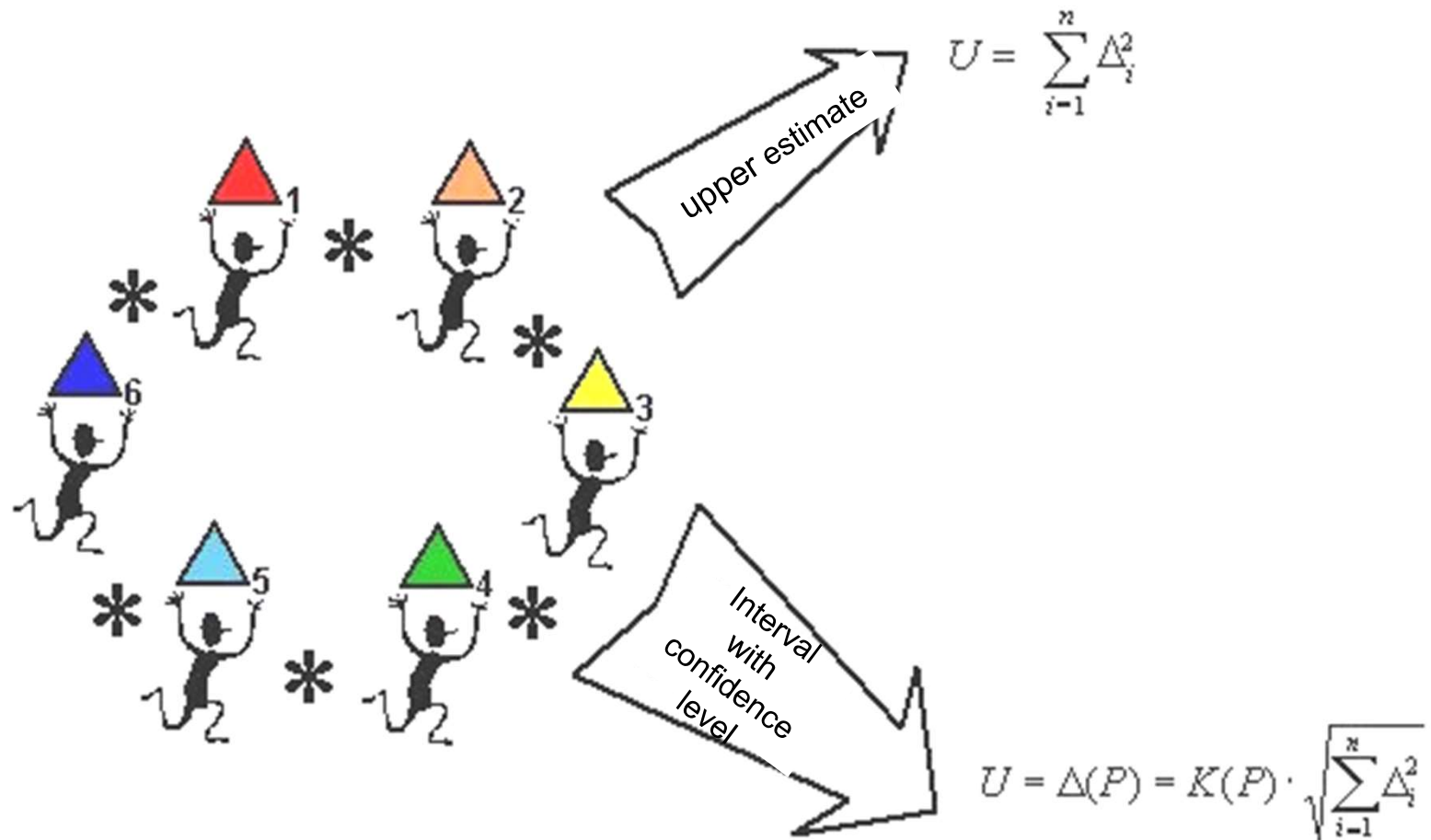


Figure 4. Estimation of expanded uncertainty with several components

Table 1. Values of the coefficients $K(P)$

<i>Trust level (confidential probability)</i>	<i>K (P) values</i>	<i>with the number</i>	<i>components n,</i>	<i>which</i>		<i>equally...</i>
	2	3	4	5	... k	mean
0.90	0.97	0.96	0.95	0.95	0.95	0.95
0.95	1.10	1.12	1.12	1.12	1.13	1.14
0.99	1.27	1.37	1.41	1.42	1.49	1.4

The result is written as:

$$x \pm U; P \quad (5)$$

Example 4

When analyzing the components of the error of a direct single measurement, the following values of the error boundaries were obtained:

$$\Delta_1 = \pm 0,10 B;$$

$$\Delta_2 = \pm 0,15 B;$$

$$\Delta_3 = \pm 0,14 B;$$

$$\Delta_4 = \pm 0,09 B;$$

$$\Delta_5 = \pm 0,13 B;$$

$$\Delta_6 = \pm 0,17 B.$$

When using the upper estimate, we obtain:

$$U = \sum_{i=1}^6 \Delta_i = 0,78 B.$$

When using the confidence level $P = 0.95$, we obtain:

$$U(P = 0,95) = 1,1 \sqrt{\sum_{i=1}^6 \Delta_i^2} = 0,36 B.$$

When using the confidence level $P = 0.99$, we obtain:

$$U(P = 0,99) = 1,4 \cdot \sqrt{\sum_{i=1}^6 \Delta_i^2} = 0,46 B.$$

Estimation $U = 0,78 V$ is overestimated (although it guarantees $P = 1$) because only in one case out of 100 will it exceed the expanded uncertainty $U = 0,46 V$. If the result is represented by the standard uncertainty u , then when assigning uniform distributions, obtain:

$$u = \frac{1}{\sqrt{3}} \sqrt{\sum_{i=1}^n \Delta_i^2} \quad (6)$$

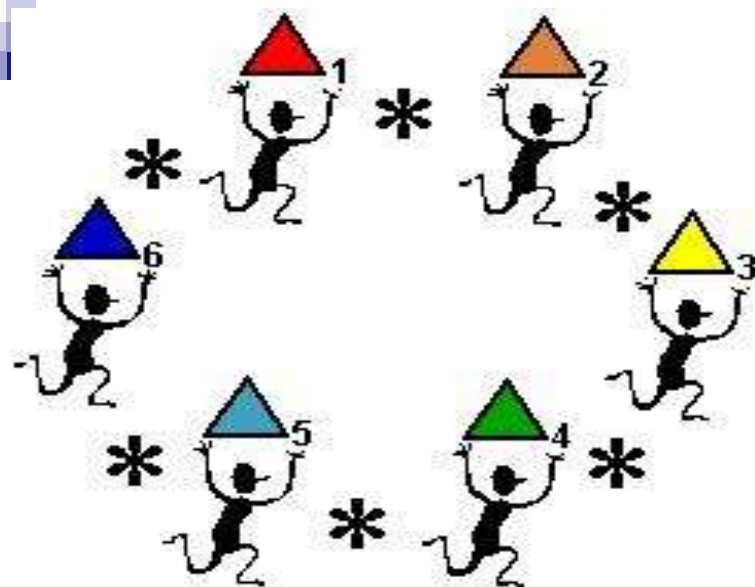


Figure 5. Estimation of standard uncertainty for several components, given by the boundaries

upper estimate $\Rightarrow u = \frac{1}{\sqrt{3}} \sqrt{\sum_{i=1}^n \Delta_i^2}$

If the components of the error are presented as boundaries of the confidence interval $\Delta_i(P)$, then the standard uncertainty can be obtained in the form $u = \sqrt{\sum_{i=1}^n (\Delta_i(P) / K_i)^2}$

where K_i is a coefficient that corresponds to the level of confidence (probability) P and the distribution of the error. If the components of the error are presented in the form of standard deviation or u_i , then the standard uncertainty is

$$u = \sqrt{\sum_{i=1}^n \sigma^2[\Delta_i]} \quad \text{or} \quad u = \sqrt{\sum_{i=1}^n u_i^2}$$

When recording the result indicating the expanded uncertainty U , use the relation

$$U = k(P) \cdot u$$

where $k(P)$ is the coverage factor, which is taken equal for $P = 0.95$, $k(P) = 2$ and for $P = 0.997$ $k(P) = 3$. The record of the result is similar to formula (5).



Example 5

Use the condition of Example 4 and find the standard uncertainty u along the given boundaries (formula (6)) **$u = 0,19 \text{ V}$** .

If we use the recommendations, then for $P = 0,997$

$$U = k(P = 0,997) \cdot u = 3 \cdot 0,19 = 0,57 \text{ B},$$

and for $P = 0,95$

$$U = k(P = 0,95) \cdot u = 2 \cdot 0,19 = 0,38 \text{ B}.$$

Discrepancies between the results of Example 4 and Example 5 for $P = 0.95$ are due to rounding errors in calculations and rounding errors of factors (coverage factors).

This proves once again that the error is known to us with limited accuracy and that the second significant figure is not entirely reliable.

7. AN EXAMPLE OF ERROR COMBINATION METHOD

Consider an example of combining errors in mass measurement. When measuring, the following measuring technique tool are used:

1 **Scales** - 1 kg –accuracy class 3;

Weights of accuracy class 3 in a standard set with nominal mass values from 10 mg to 500 g.

The measurement result is determined using the ratio: $m = \sum_{i=1}^n m_{\Gamma i} + m_{\varphi} + a$

where $m_{\Gamma i}$ (*Weights*) is the nominal value of the mass of the i-th weight, m_{φ} is the scale reading, a is the correction for the action of the aerostatic force, which is calculated by the formula

$$a = \left(\sum_{i=1}^n m_{\Gamma i} + m_i \right) \cdot \rho_e \cdot \frac{\rho_{\Gamma} - \rho_{\mathcal{A}}}{\rho_{\Gamma} \cdot \rho_{\mathcal{A}}}$$

where $\rho_e = (1.20 \pm 0.12) \text{ kg/m}^3$ - air density; $\rho_{\mathcal{A}} = (3.2 \pm 0.2) \cdot 10^3 \text{ kg/m}^3$ - density of the alloy from which the parts that are weighed are made; $\rho_{\Gamma} = 8 \cdot 10^3 \text{ kg/m}^3$ - density (reduced) weights. When calculating the errors, the following passport data of the scales are taken into account: - scale division value 10 mg; - the error in measuring the mass on the scale $\Delta_{\mathcal{U}}(m)$ does not exceed half the scale; - error due to asymmetrical beams $\Delta_{\mathcal{H}}(m)$ 10 mg; - error due to the permutation of weights $\Delta_n(m)$ 5 mg; - variation of readings at 10 measurements $\Delta_e(m)$ 10 mg.

The error in measuring the mass is the sum of the error of the set of weights that is used, $\Delta(m_r)$, the weighing error $\Delta(m_\varphi)$ and the correction error $\Delta(a)$. The error of a set of weights

$$\Delta(m_r) = \sum_{i=1}^n \Delta(m_i),$$

where $\Delta(m_i)$ - errors of individual set weights, which are found from tables in accordance with the nominal value and accuracy class of the weights.

The weighing error includes $\Delta(m_\varphi) = \Delta_{\text{w}}(m) + \Delta_{\text{a}}(m) + \Delta_{\text{n}}(m) + \Delta_{\text{e}}(m)$

The correction error includes the components $\Delta(a) = \Delta_B(a) + \Delta_D(a) + \Delta_m(a)$

where $\Delta_B(a)$ is the component of the error due to the inaccuracy of the air density value; $\Delta_D(a)$ - component of the correction error due to the inaccuracy of the density value of the part; $\Delta_m(a)$ - the component of the correction error due to the inaccuracy of the mass value. The components of the correction error can be obtained from the following relations:

$$\Delta_B(a) = \frac{\partial a}{\partial \rho_e} \cdot \Delta(\rho_e) = \frac{m(\rho_r - \rho_D)}{\rho_r \cdot \rho_D} \cdot \Delta(\rho_e)$$

$$\Delta_D(a) = \frac{\partial a}{\partial \rho_D} \cdot \Delta(\rho_D) = -\frac{m \cdot \rho_e}{\rho_D^2} \cdot \Delta(\rho_D)$$

$$\Delta_m(a) = \frac{\partial a}{\partial m} \cdot \Delta(m) = \frac{\rho_e \cdot (\rho_r - \rho_D)}{\rho_r \cdot \rho_D} \cdot \Delta(m)$$

wherein $\Delta(\rho_e)$ - air density error; $\Delta(\rho_D)$ - details of the error density; $\Delta(m)$ - mass measurement error.

Let us assume that a counterbalancing kettlebell set is $500 \text{ g} + 100 \text{ g} + 10 \text{ g} + 1 \text{ g} + 100 \text{ mg} + 10 \text{ mg} = 611,11 \text{ g}$.

The limits of the kettlebell set error interval are $\Delta(m_r) = \pm 8 \text{ mg} \pm 4 \text{ mg} \pm 1,2 \text{ mg} \pm 0,4 \pm \text{mg} \pm 0,1 \text{ mg} = \pm 13,8 \text{ mg}$.

If the balance is balanced, the measurement error is $\Delta(m_p) = \pm 10 \pm 10 \pm 5 = \pm 25 \text{ mg}$.

Amendment is $a = 137 \text{ mg}$, and its error

$$\Delta_e(a) = \pm 14 \text{ mg}, \Delta_d(a) = \pm 14 \text{ mg}, \Delta_m(a) = \pm 18 \cdot 10^{-3} \text{ mg}.$$

Let us represent the measurement result with the uncertainty characteristic in the form of the boundaries of the interval with the probability $P = 1$. Then the boundaries of the intervals of individual errors are combined in accordance with

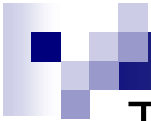
$x \pm \Delta$ and the corrected result, in accordance with $m = \sum_{i=1}^n m_{r_i} + m_p + a$, is represented, taking into account the uncertainty, in the form

$$m = (611,25 \quad 0,07) \text{ g}.$$

Since such an estimate of uncertainty is overestimated, the measurement result can be presented with an indication of the expanded uncertainty at a confidence level of $P = 0,95$. For this, we use expression (9), assuming $P = 0,95$ and $K(P) = 1,1$. Presentation of the result with expanded uncertainty

$$m = (611,25 \quad 0,03) \text{ g}; P = 0,95.$$

The measurement result can be presented with the combined standard uncertainty found from the following:



The errors of the kettlebell set $\Delta(m_r)$ are attributed to a trapezoidal distribution with $\beta = 0.5$. Weighing errors $\Delta(m_\varphi)$ are attributed to a triangular distribution. Errors in introducing the correction $\Delta_\varepsilon(a)$ and $\Delta_d(a)$ are assigned uniform distributions. We neglect the error $\Delta_m(a)$ due to its smallness.

Accordingly, we obtain variance estimates.

$$u^2(m_r) = \frac{13,8^2 \cdot 1,25}{6} = 39,68 \text{ (Mg)}^2 ; \quad u^2(m_\varphi) = \frac{25^2}{6} = 104,17 \text{ (Mg)}^2 ; \quad u_\varepsilon^2(a) = u_d^2(a) = \frac{14^2}{3} = 65,33 \text{ (Mg)}^2$$

Then the standard uncertainty is

$$u(m) = \sqrt{u^2(m_r) + u^2(m_\varphi) + u_\varepsilon^2(a) + u_d^2(a)} = \sqrt{274,5} = 16,5 \text{ Mg}$$

Then the measurement result can be represented as

$$m = 611.25 \text{ g}; u(m) = 0.02 \text{ g},$$

where **$u(m)$** is the standard uncertainty

The expanded uncertainty at a confidence level of $P = 0.95$ can be obtained as

$$U = k * u(m) = 2 * 16.5 = 33 \text{ mg}.$$

Then the measurement result can be represented as

$$m = (611.25 \pm 0.03) \text{ g}; P = 0.95,$$

which coincides with that obtained earlier $U = \Delta(P) = K(P) \cdot \sqrt{\sum_{i=1}^n \Delta_i^2}$



Conclusion:

Say that the complexity of a measurement lies in its apparent simplicity.

Indeed, what could be simpler than a direct single measurement?

But it turns out that in such a measurement there can be many sources of error, and if you forget about some, then you can greatly underestimate the total error.

Obviously, in the ideal case, one should take into account all possible sources of error and combine them correctly, presenting the result with its uncertainty.



**Thank you for your
attention!**






The course
theme:

Methods and means of measurement

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Lecture 12
**DIRECT MULTIPLE
MEASUREMENTS OF
QUASIDETERMINAL
VALUES**



The purpose of this lecture is to study the statistical processing of the results of direct multiple measurements, as a result of which the random component of the error decreases and the measurement accuracy increases.

Lecture plan

1. *The result of direct multiple measurements of a quasi-deterministic quantity.*
2. *Censoring the sample with a normal distribution of measurement results.*
3. *Checking the normal distribution of measurement results.*
4. *Methodology for evaluating the result with uncertainty in direct multiple measurements without grouping.*
5. *Criteria for negligible error.*
6. *An example of processing the results of direct multiple measurements without grouping.*

1. RESULT OF DIRECT MULTIPLE MEASUREMENTS OF A QUASID-DETERMINED VALUE

A **quasi-deterministic** measurable quantity is a measurable quantity that corresponds to a specific model. As an example of multiple measurements of a quasi-deterministic quantity, one can cite multiple measurements of the amplitude of a sinusoidal signal (the form of the signal is determined, the measured quantity is a signal parameter, amplitude). If the random error is significant, then the individual results x_i with multiple measurements will be random, i.e.

$$x_i = x_{i\text{uom}} + \Delta_s + \overset{0}{\Delta_i}$$

Before statistical processing, it is necessary to detect and eliminate systematic errors, since during statistical processing they cannot be reduced. The next task is to obtain a measurement result for a series of multiple measurements x_i , which will be as close as possible to the true value. Such a result can be obtained if the model of the random error is known, i.e., the distribution. If the distribution of the random error is normal, then such an estimate of the measured value will be the arithmetic mean.

$$x_{\text{quas}} = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

But the arithmetic mean is very sensitive to deviations from the accepted model, which appear in the form of misses (measurement results that have an excessive error), “heavy tails” of the distribution (when the probability of large errors exceeds the allowable for a normal distribution).



2. CENSORSHIP OF THE SAMPLE WITH A NORMAL DISTRIBUTION OF THE MEASUREMENT RESULTS

Criteria for censoring with a normal distribution of measurement results (rules for assessing the abnormality of measurement results) are classified according to the availability of a priori information in the form of a known general average and general standard deviation into:

- criterion for evaluating the abnormality of measurement results with an unknown general average and unknown general standard deviation;
- criterion for evaluating the abnormality of measurement results with an unknown general average and a known general standard deviation;
- criterion for evaluating the abnormality of measurement results with a known general average and known general standard deviation.

When processing a number of direct measurements, the most common situation is when the general mean and standard deviation are unknown. Then, for an ordered sample of measurement results $x_1 \leq x_2 \leq x_3 \dots \leq x_n$, the sample mean and sample standard deviation \bar{x} are calculated using the formulas

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}}$$

To assess whether x_n or x_1 belongs to a given normal population and to make a decision about excluding or leaving x_n (x_1) in the sample, find the ratio

$$U_n = \frac{x_n - \bar{x}}{S} \quad \text{or} \quad U_1 = \frac{\bar{x} - x_1}{S}$$

The result is compared with the value of the coefficient h from the **table 1** (fragment) for a given sample size n and the accepted probability.

Sample size	Limit values	with probability
	0.050	0.025
1	2	3
3	1.15	1.15
4	1.46	1.48

If $U_n \geq h$ ($U_1 \geq h$), then the suspected abnormality result of the measurement is excluded, otherwise it is left.

If n is 20, then a **table 2** (fragment) is used

Sample size n	Limit	h values	with probability
	0.050	0.025	0.010
3	21,786	70,822	70,822
4	7,217	10,117	15.918

3. VERIFICATION OF THE NORMALITY OF THE DISTRIBUTION OF THE MEASUREMENT RESULTS

To check the normality of the empirical distribution (checking the agreement of the empirical distribution with the theoretical normal), the Kolmogorov criteria, criterion χ^2 , criterion ω^2 , criterion W , composite criterion are used.

Kolmogorov's criterion is used for a sample size greater than 100, χ^2 - for a sample size greater than 200, ω^2 - for a sample size greater than 50. W criterion is used for a sample size from 3 to 50, a composite criterion for a sample size from 15 to 50.

When processing results of direct multiple measurements, a composite criterion is used. The composite criterion consists of two stages. At the first stage, the ratio

$$\tilde{d} = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n \cdot S_*},$$

$$S_* = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

where S^* - biased standard deviation estimate

We can assume that the measurement results are normally distributed if

$$d_{1-q_1/2} < \tilde{d} \leq d_{q_1/2}$$

where $d_{1-q_1/2}$ and $d_{q_1/2}$ are the quantiles of the distribution, which are obtained from the table in accordance with the chosen significance level q_1 .

At the second stage of verification, it is considered that the measurement results belong to the normal distribution, if no more than m differences $(x_i - \bar{x})$ exceed the value $z_{p/2} \cdot S$,

where S is the sample standard deviation,

$z_{p/2}$ is the upper quantile of the distribution of the normalized Laplace function, which corresponds to the probability $P/2$.

P - values are obtained from a table 3 according to the chosen significance level q_2 and sample size n .

Table 3. P values for calculating $z_{p/2}$

n	m	q2 * 100%			n	m	q2 * 100%		
		1%	2%	5%			1%	2%	5%
10	1	0.98	0.98	0.96	24 ... 27	2	0.98	0.98	0.97
11 ... 14	1	0.99	0.98	0.97	28 ... 32	2	0.99	0.98	0.98
15 ... 20	1	0.99	0.99	0.98	33 ... 35	2	0.99	0.98	0.98
21 ... 22	2	0.98	0.97	0.96	36 ... 49	2	0.99	0.99	0.98
23	2	0.98	0.96						

If, when checking the normal distribution for stage 1, the selected significance level was q_1 , and for stage 2 - q_2 , the resulting significance level of the composite criterion $q \leq q_1 + q_2$.

If at least one of the steps is not fulfilled, it is considered that the distribution of the measurement results does not correspond to the normal one.



4. METHOD FOR EVALUATING THE RESULT WITH DIRECT MULTIPLE MEASUREMENTS WITHOUT GROUPING

The procedure for evaluating the result of direct multiple measurements can be divided into the following stages:

- elimination of known systematic errors;
- calculation of the arithmetic mean of the corrected measurement results, which is taken as an estimate of the measured value;
- calculation of sample standard deviation;
- censoring the sample, provided that it is considered a priori distributed normally;
- testing the hypothesis regarding the normality of the sample;
- calculation of RMS estimates of random and non-excluded systematic errors, calculation of standard combined uncertainty;
- presentation of the measurement result.

When performing the main stages, the following recommendations should be taken into account:

- when testing the hypothesis regarding the normality of the distribution of the sample, the significance level is selected within the range of values $0,02 \dots 0,1$;
- to determine the confidence limits of the measurement result error, select the probability $0,95$;
- if the measurement cannot be repeated, except for the boundaries with the probability $P = 0,95$, it is possible to indicate the boundaries with the probability $P = 0,99$;
- for measurements in medicine and ecology, it is allowed to take an even higher confidence probability instead of $P = 0,99$.

The sequence of evaluating the result for direct multiple measurements without grouping is given below:

4 .1. The corrected measurement results are entered in table 4:

$N_{\text{д}}$	x_i	$v_i = x_i - \bar{x}$	v_i^2
1	x_1	v_1	v_1^2
n	x_n	v_n	v_n^2
	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$	$\sum_{i=1}^n v_i = 0$	$\sum_{i=1}^n v_i^2$

4.2. Find the arithmetic mean - the result of multiple measurements $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$

4.3. Calculate random deviations $v_i = x_i - \bar{x}$, the results are entered in table 1.

4.4. Find the sample standard deviation. $S = \sqrt{\sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1}}$

4.5. The sample is censored. If there is an abnormal result, it is excluded and the calculations begin from clause 4.1.

4.6. The sample is checked for normality.

4.7. Find an estimate of the standard deviation of the random component of the measurement result error

$$\sigma[\Delta] = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n(n-1)}}$$

4.8. Calculate the characteristics of the non-excluded systematic error of the measurement result. These include a methodical error, an error due to interaction, leaving errors of the measuring instrument. Permanent systematic errors are eliminated by introducing a correction, the errors of introducing the correction are estimated. These components of the error are type B components, i.e., to estimate the standard uncertainty, one must use information about possible distributions or assign distributions based on an upper estimate. If uniform distributions are assigned to all components, then

$$u_s = \sigma_m[\Delta_s] = \sqrt{\sum_{i=1}^m u_i^2} = \frac{1}{\sqrt{3}} \sqrt{\sum_{i=1}^m \Delta_{si}^2} \quad \text{where } \mathbf{m} \text{ is the number of error components.}$$

4.9. To represent the result, you can use the standard uncertainty u equal to

$$u = \sqrt{\sigma^2[\Delta] + u_s^2}$$

Then the measurement result has the form $x = \bar{x}$; u ; number of measurements n .

4.9.1. If $\sigma[\Delta]/u_s > 3$ (the requirement of the criterion of negligible error is met, which will be considered below), then the measurement result can be presented in the form $x = \bar{x}$; $\sigma[\Delta]$; the distribution of uncertainty is normal with the number of degrees of freedom $\nu = n - 1$.

4.9.2. If $\sigma[\Delta]/u_s < 1/3$, then the measurement result can be represented as $x = \bar{x}$; u_s ; number of measurements n .

4.10. To represent the result with expanded uncertainty, it is necessary to find the coverage factor k at a given level of confidence P . In the standard, the following method for estimating the factor k is proposed

$k = \frac{t \cdot \sigma[\Delta] + U_s}{\sigma[\Delta] + u_s}$, where t is the Student's coefficient, which is determined from the table 5 (fragment) based on the confidence probability P and the number ν degrees of freedom, U_s is the expanded uncertainty of non-excluded systematic errors, which is calculated as for direct single measurements.

	P			
$\nu - 1$	0,90	0,95	0,98	0,99
1	6,314	12,706	31,821	63,657



When assigning uniform distributions and the level of confidence $P = 0.95$, U_s is equal to

$$U_s = 1.1 \sqrt{\sum_{i=1}^m \Delta_{si}^2}$$

Then the measurement result is presented in the form:

$$x = \bar{x} \pm U; P$$

where $U = k \cdot u$.

When meeting the requirements 4.9.1. the measurement result using expanded uncertainty is presented as:

$$x = \bar{x} \pm t \cdot \tilde{\sigma}[\Delta]; P$$

If the requirements 4.9.2 are met the measurement result using expanded uncertainty is presented as (for the confidence level P):

$$x = \bar{x} \pm U_s; P$$

5. CRITERION OF NON-ERROR

When summing the errors, the procedure for finding the composition of distributions and the coverage factor can be simplified by using the criterion of negligible error. The criterion of negligible error gives an answer to the question, what kind of error can be discarded before summation due to its smallness. Standard deviation of the total error $\sigma [\Delta_{sum}]$ is $\sigma[\Delta_{sum}] = \sqrt{\sigma^2[\Delta_1] + \sigma^2[\Delta_2]}$

When the standard deviation (SD) error rounding $\sigma [\Delta_{sum}]$ in SD is no more than two significant digits. The error of rounding off to two significant figures does not exceed 5%. Thus, a small error can be discarded before stacking if it changes by less than 5%. That is, if $\sigma [\Delta_2]$ is small

$$\sigma^2[\Delta_{sum}] = \sigma^2[\Delta_1] + \sigma^2[\Delta_2] \cong \sigma^2[\Delta_1]; \quad \frac{\sigma[\Delta_{sum}]}{\sigma[\Delta_1]} \leq 1.05;$$

$$\frac{\sigma^2[\Delta_{sum}]}{\sigma^2[\Delta_1]} \leq (1.05)^2 \quad \text{or} \quad 1 + \frac{\sigma^2[\Delta_2]}{\sigma^2[\Delta_1]} \leq 1.1 \quad \sigma^2[\Delta_2] \leq 0.1 \cdot \sigma^2[\Delta_1]; \quad \sigma[\Delta_2] \leq 0.3 \cdot \sigma[\Delta_1]$$

Thus, if the standard deviation of a smaller error value is three times less than a larger error value, it can be neglected. A group of errors is discarded if their sum is less than one third of the maximum error.

$$\sqrt{\sum_{i=2}^l \sigma^2[\Delta_i]} \leq 0.3 \cdot \sigma[\Delta_1]$$

6. EXAMPLE OF PROCESSING THE RESULTS OF DIRECT MULTIPLE MEASUREMENTS WITHOUT GROUPING.

The results of direct multiple voltage measurements are given in table. 6.

Table 6. An example of processing direct multiple measurements

$x_i (B)$	<i>The number of measurements n_i equal to x_i</i>	$v_i (B)$	$v_i^2 (B)$
17.6	3	-0.2	0.04
17.7	10	-0.1	0.01
17.8	16	0	0
17.9	10	0.1	0.01
18.0	3	0.2	0.04
$\bar{x} = 17.800$	$\sum n_i = n = 42$	$\sum v_i = 0$	$\sum v_i^2 = 0.44$

A voltmeter of accuracy class 0.1 with the final value of the measuring range of 30 V was used for the measurement. The additional error arising during the measurement did not exceed the basic one. The resistance of the voltmeter $R_V \geq 0.5 M\Omega$, the load resistance $R_H = 1 k\Omega$, the resistance of the circuit $R_H = 100 Ohm$ (Fig. 1).

Measurement result - arithmetic mean $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = 17,800 B$

The values of random deviations are given in table. 1. The sum of random deviations is zero. $(-0.2) \cdot 3 + (-0.1) \cdot 10 + 0 \cdot 16 + 0.1 \cdot 10 + 0.2 \cdot 3 = 0$.

Sample standard deviation $S = 0.104 V$.

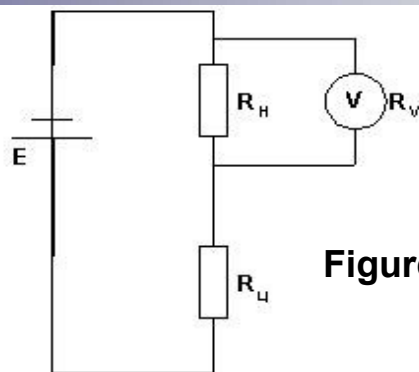


Figure 1. Scheme for switching on a voltmeter.

When censoring the sample, we select a criterion with an unknown general mean and unknown general standard deviation. Sample table 2 is ranked, so the extreme members of the sample are suspected of abnormality. Find the largest value

$$U_n = \frac{x_n - \bar{x}}{S} = \frac{0.2}{0.104} = 1.96$$

and compare it with the value of h from the table (Table 7, fragment) for the significance level $\alpha = 0.01$ and the sample size $n = 42$. The value $h = 3.48$. If $U_n < h$, we conclude that there are no abnormal results.

Sample size n	Limit	h values	at	probabilities
	0.050	0.010	0.005	0.001
1	1,645	2,236	2,576	3.090
2	1,955	2,575	2.807	3.290
3	2.121	2,712	2.935	3.403
4	2,234	2.806	3.023	3.481

To check the normality, we choose a composite criterion. According to the first part of the criterion, we calculate the value $S^* = 0.102 \text{ V}$ and the value $\tilde{\alpha}$



$$\tilde{d} = \frac{\sum_{i=1}^n |v_i|}{n \cdot S_*} = \frac{0.2 \cdot 3 \cdot 2 + 0.1 \cdot 10 \cdot 2}{42 \cdot 0.102} = \frac{3.2}{42 \cdot 0.102} = 0.7448$$

We choose the significance level $q_1 = 0.1$ and from the table (Table 8, fragment) we obtain the quantiles $d_{1-q_1/2} = 0.7470$ and $d_{q_1/2} = 0.8722$. The value \tilde{d} does not fall within the interval between these limits.

n	$q_{1/2} * 100\%$		$(1-q_{1/2}) * 100\%$	
	1%	5%	95%	99%
16	9137	8884	7236	6829
21	9001	8768	7304	6950
26	8901	8686	7360	7040
31	8826	8625	7404	7110
36	8769	8578	7440	7167

Therefore, we reduce the level of significance, which in turn leads to the expansion of boundaries. If $q_1 = 0.02$, then $d_{1-q_1/2} = 0.7216$ and $d_{q_1/2} = 0.8722$. The value \tilde{d} is between these limits and, thus, you can proceed to checking the second part of the criterion. We choose $q_2 = 0.02$ and get $P = 0.99$, $m = 2$. Then $z_{P/2} = 2.6$ (Table 9, fragment) and the boundary value $\Delta_r = z_{P/2} \cdot S = 0.26 B$.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.0	0.00000	1.4	0.41924	2.8	0.49744
0.1	0.03983	1.5	0.43319	2.9	0.49813
0.2	0.07926	1.6	0.44520	3.0	0.49865
0.3	0.11791	1.7	0.45543	3.1	0.49903
0.4	0.15542	1.8	0.46407	3.2	0.49931
0.5	0.19146	1.9	0.47128	3.3	0.49952
0.6	0.22804	2.0	0.47725	3.4	0.49966

None of the random variation does not exceed the value of Δ_T . Therefore, the hypothesis of normal distribution is accepted according to a composite criterion with a significance level of $q = 0.04$. We calculate the estimate of the standard deviation of the random component of the error $\hat{\sigma}[\Delta] = \frac{S}{\sqrt{n}} = 0.0160 \text{ B}$

We calculate the characteristics of the non-excluded systematic component of the error. For this, we calculate the components of the error. The relative error in the interaction of the voltmeter with the object is calculated using the circuit in Fig. 2: $\delta_e = \frac{U'}{U} - 1$, where U is the voltage drop across the load R_H before the voltmeter is turned on; U' - voltage drop across the load R_H after turning on the voltmeter.

From where:
$$U = \frac{E \cdot R_H}{R_H + R_{II}}; \quad U' = \frac{E \cdot \frac{R_H \cdot R_{II}}{R_H + R_{II}}}{R_{II} + \frac{R_H \cdot R_{II}}{R_H + R_{II}}} = \frac{E \cdot R_H \cdot R_V}{R_H \cdot R_{II} + R_{II} \cdot R_V + R_H \cdot R_V}$$

$$\delta_e = \frac{R_V (R_H + R_{II})}{R_H \cdot R_{II} + R_{II} \cdot R_V + R_H \cdot R_V} - 1 = \frac{-R_H \cdot R_{II}}{R_H \cdot R_{II} + R_{II} \cdot R_V + R_H \cdot R_V}$$

Substituting into the expression for the values of the resistances R_c , R_H , R_V , we get $\delta_e = -\frac{10^5}{5.5 \cdot 10^8} \cdot 100\% = -0.018\%$. Since the error is systematic, we exclude it by introducing a correction $a = -\Delta_e = -\delta_e \cdot \bar{x} = 0.018 \cdot 10^{-2} \cdot 17.80 = 0.0032 \text{ B}$

The error due to the interaction is small and therefore the error in introducing the correction can be omitted. Corrected measurement result $\bar{x}_{ucn} = \bar{x} + a = 17.803 \text{ B}$

The basic error of the measuring instrument is $\Delta_o = \pm \gamma \cdot x_n = \pm 0.1 \cdot 10^{-2} \cdot 30 = \pm 0.03 \text{ B}$

Additional error is equal to basic $\Delta_b = \pm 0.03 \text{ B}$

Then the standard uncertainty caused by non-excluded systematic errors is

$$u_s = \frac{1}{\sqrt{3}} \sqrt{0.03^2 + 0.03^2} = 0.0245 \text{ B}$$

The standard uncertainty of the measurement result is $u = \sqrt{\sigma^2[\Delta] + u_s^2} = 0.0293 \text{ B}$

Recording the result with standard uncertainty:

$x = 17.803 \text{ V}$; $u = 0.029 \text{ V}$; number of measurements $n = 42$.

To record the result with expanded uncertainty, we set the confidence level P and calculate the coverage factor

$$k = \frac{t \cdot \sigma[\Delta] + U_s(P)}{\sigma[\Delta] + u_s}$$

where $U_s(P) = 1.1 \sqrt{\Delta_b^2 + \Delta_b^2} = 0.047 \text{ B}$

Then $k = 1.954$ $U = ku = 0.057 \text{ V}$.

Writing the result:

$$x = (17,80 \pm 0,06) \text{ V}; P = 0,95.$$



Conclusion:

By studying the material of this lecture, you have seen that the standard uncertainty of the arithmetic mean decreases by $\frac{1}{\sqrt{n}}$ when performing n measurements.

But this does not mean that if you are patient and perform a huge number of measurements, you can infinitely reduce the error without improving the measuring instrument.

An increase in n leads to a decrease in only the random component of the measurement.

Non-excluded systematic errors remain unchanged. It is clear from this that there is no great benefit in further reducing the random component of the error if it becomes less than the systematic component.

All this only confirms the fact that, in practice, a significant reduction in error requires improvements both in the measuring instrument and in the method of processing the measurement results.



**Thank you for your
attention!**

